Lecture Notes: Entry and Product Positioning

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Models of entry and product positioning

Motivation:

- Static demand and supply analyses treat the set of available products as given
- This raises two obvious questions:
  1. What determines the set of available products?
  2. How would counterfactual analyses change if we allowed market structure to be endogenous? in the short-run or the long-run?
- Answering these questions requires models that can capture the roles of (at least):
  - market structure, technology and strategic interactions

The objective of the empirical game literature is to provide tractable econometrics models that can be used to study the structure of markets (i.e. number of firms, and characteristics of products). In the first lecture we will study "long-run" models of market structure (i.e. static entry games), and in the next two lectures we will talk about the estimation of dynamic games.

Examples of research questions:

1. What determines the equilibrium number of firms in a market?
    - Is the number of firms be optimal? If not, what is role for licensing or entry subsidies?
    - Will the number of firms increase with market size?
    - What is the effect of cyclical variation in market size/demand on market structure and competitiveness
2. When can firms act to distort market structure?
    - entry deterrence, accommodation and predation
    - When can such strategies be profitable or credible? How can we detect/test distortions?
3. How environmental regulations or taxes affect the variety of products offered? What are the welfare consequences?
4. How might patent reform affect new drug research and entry?
5. Will new entry deal with competition concerns after a merger? Or will the merged firm increase the number of products that it offers or differentiate them?
6. What are optimal rules (or subsidies) for auctions in the presence of an entry margin?

Back to basics: Estimation to two-players entry games (Tamer 2003)

- Consider the following 2x2 entry game:

	$y_2 = 0$	$y_2 = 1$
$y_1 = 0$	(0,0)	$(0, x_2\beta_2 + u_2)$
$y_1 = 1$	$(x_1\beta 1 + u_1, 0)$	$(x_1\beta_1 + \Delta_1 - u_1, x_2\beta_2 + \Delta_2 + u_2)$

  where $\Delta_1 < 0$ and $\Delta_2 < 0$.

- **Assumptions:** (i) (u_1, u_2) are unobserved to the econometrician, but commonly observed by both players, (ii) firms simultaneously and non-cooperatively choose their actions, and (iii) (y_1, y_2) are determined by a Nash equilibrium solution.

- Indirect utility and decision rules for every $j = 0, 1$:

$$v_j = x_j\beta_j + y_{-j}\Delta_j + u_j$$
$$y_j = \begin{cases} 1 & \text{If } v_j \geq 0 \\ 0 & \text{Else.} \end{cases}$$

- The joint likelihood of observing (y_1, y_2) is determined by a series of bivariate probabilities:

$$\begin{aligned}
\Pr[(0,0)|x] &= \Pr(u_1 < -x_1\beta_1; u_2 < -x_2\beta_2) \\
\Pr[(0,1)|x] &= \Pr(u_1 < -x_1\beta_1 - \Delta_1; u_2 \geq -x_2\beta_2 - \Delta_2) \\
\Pr[(1,0)|x] &= \Pr(u_1 \geq -x_1\beta_1 - \Delta_1; u_2 < -x_2\beta_2 - \Delta_2) \\
\Pr[(1,1)|x] &= \Pr(u_1 \geq -x_1\beta_1 - \Delta_1; u_2 \geq -x_2\beta_2 - \Delta_2)
\end{aligned}$$

- Multiple equilibria: The model has multiple Nash equilibrium in the monopoly region $(0, 1)$ or $(1, 0)$.

- This lead to an *incoherent* econometric model:

$$\Pr[(0,0)|x] + \Pr[(0,1)|x] + \Pr[(1,0)|x] + \Pr[(1,1)|x] > 1$$

- Solution? Need to impose some restrictions on $\Pr[(0,1)|x]$ and $\Pr[(1,0)|x]$
- In particular, lets define $\Pr((1,0)|x)$:

$$\Pr[(1,0)|x] = 1 - \Pr[(0,0)|x] - \Pr[(0,1)|x] + \Pr[(1,1)|x]$$

- At best, the theory can impose bounds on $\Pr[(0,1)|x]$:

$$L(x,\beta) \leq \Pr[(0,1)|x] \leq H(x,\beta)$$

Incomplete model

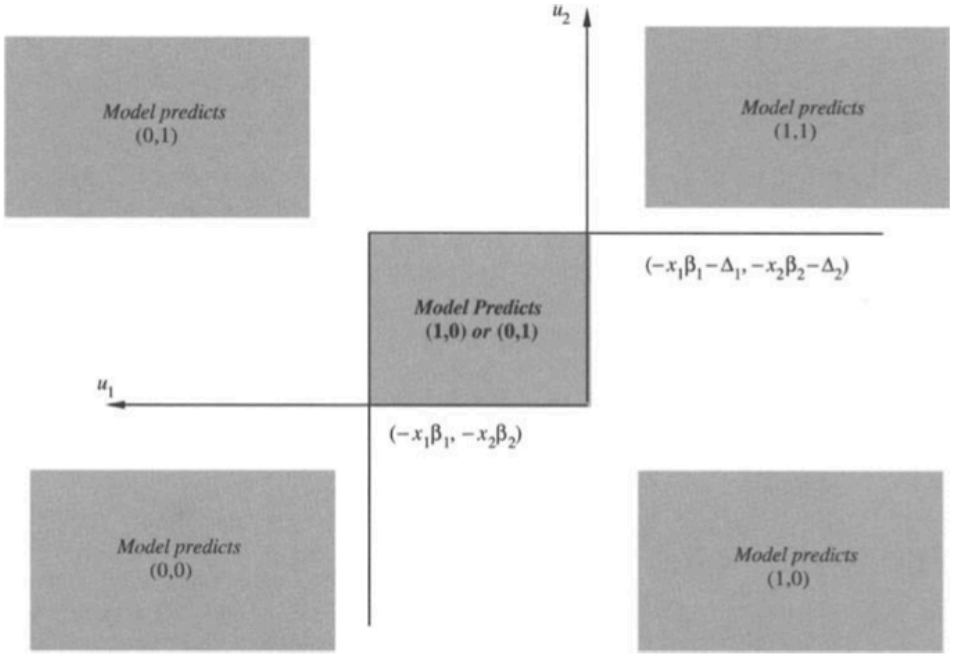


FIGURE 1
Incomplete model with multiple equilibria

Complete model: Maximum probability of $(0, 1)$

$$H(x, \beta) = \Pr(u_1 < -x_1\beta_1 - \Delta_1; u_2 \geq -x_2\beta_2)$$

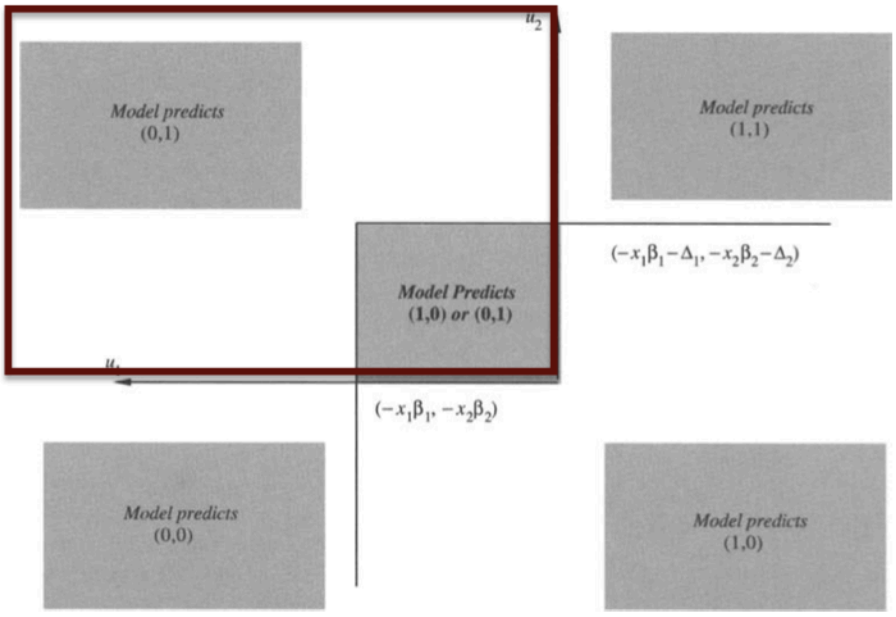


FIGURE 1
Incomplete model with multiple equilibria

Complete model: Minimum probability of $(0, 1)$

$$L(x, \beta) = \Pr(u_1 < -x_1\beta_1 - \Delta_1; u_2 \geq -x_2\beta_2 - \Delta_2)$$
$$+ \Pr(u_1 < -x_1\beta_1; -x_2\beta_2 \leq u_2 < -x_2\beta_2 - \Delta_2$$

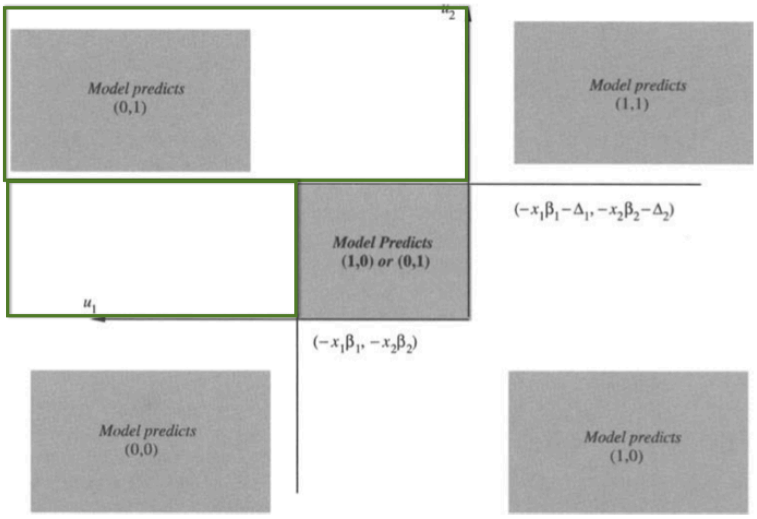


FIGURE 1
Incomplete model with multiple equilibria

Example of solutions used in the literature:

- No heterogeneity assumption: $u_1 = u_2 = u$.
  - This was first proposed by Bresnahan and Reiss (1990)
  - If firms are symmetric, the model predicts unique **aggregate** outcomes

$$\begin{aligned}
\Pr(n = 0|x, \beta) &= \Pr(u < -x\beta) \\
\Pr(n = 1|x, \beta) &= \Pr(-x\beta < u < -x\beta - \Delta) \\
\Pr(n = 2|x, \beta) &= \Pr(-x\beta - \Delta < u)
\end{aligned}$$

    If $u \sim N(0, 1)$ leads to a very simple ordered-probit model.

- Heterogeneity model with partial likelihood
  - The model above predicts unique probability for three outcomes: $(0, 0)$, $(1, 1)$, and $(0, 1)$ OR $(1, 0)$
  - The log-likelihood function can thus be re-written as:

$$\begin{aligned}
L(y, x, \beta) \;=\; & \sum_t (1 - y_{1t})(1 - y_{2t}) \log \Pr\left[(0, 0)|x\right] + y_{1t} y_{2t} \log \Pr\left[(1, 1)|x\right] \\
& + \left((1 - y_{1t})y_{2t} + y_{2t}(1 - y_{2t})\right) \left(1 - \Pr\left[(0, 0)|x\right] - \Pr\left[(1, 1)|x\right]\right)
\end{aligned}$$

  - While this approach yields point estimate of β, it is not the most efficient estimator since it ignores variation that generates $(0, 1)$ and $(1, 0)$. Also, it is not generalizable to more than 2 firms.
  - Tamer (2003) and Ciliberto and Tamer (2009) propose an approach that relies of new development in the econometrics of *incomplete* models

- Equilibrium selection rules (Berry 1992, Mazzeo 2001): Sequential move
  - If the game is played sequentially, the model produces a unique subgame perfect equilibrium.
  - Examples: Order of entry is determined by firms profitability, or incumbency
- Moment inequality approach: Pakes, Porter, Ho, and Ishii (2006) (among others)
  - Instead of imposing a selection rule and/or distributional assumptions, this approach tries to compute bounds on the parameter values that satisfy the equilibrium conditions

Application 1: Entry and Competition in Concentrated Markets (Bresnahan and Reiss 1991)

- **Question:** What can we learn about **industry conduct** from studying the relationship between N and market size?
- Demand *entry threshold*: Market size required to support N firms under free-entry conditions.
- **Contribution:** Propose a method to estimate the *entry thresholds* of an industry, and show that these provide *scale-free* measures of the effect of entry on market conduct.
- **Road-map:**
  - Develop a long-run model of an industry market-structure
  - Show how the free-entry condition leads to an ordered-probit estimator of variable profits and fixed-costs
  - **Case studies:** Dentist, physicians, tire stores, plumbers, etc.
  - Additional references: Bresnahan and Reiss (1987), Bresnahan and Reiss (1990).

- **Assumptions:**
  - *Homogenous firms:* Firms have symmetric cost and revenue functions
  - *Complete information:* Common knowledge about the long-run profit of being in the industry
  - *Steady-state:* The industry is mature and the observed market-structure is stable.
- **Profits:**

$$\Pi_n(S_n) = [P_n - AVC_n(q_n, W)]\, d(Z, P_n)\frac{S}{n} - F_n$$

  where,
  - n is the number of firms active
  - $d(Z, P_n)$ is the demand *per consumer* in markets with characteristics Z
  - S is the market-size
  - P_n is the equilibrium price
  - $AVC(q_n, W)$ and F are variable and fixed-costs
- **Note 1:** Variable and fixed-costs are indexed by n to allow the possibility that later entrants have higher variable or fixed-costs (e.g. barrier to entry)
- **Note 2:** This profit function should be thought of as the profit of the *marginal entrant*

- **Free-entry:** The industry market-structure is such that:

$$\Pi_{n^*}(S_{n^*}) \geq 0 \text{ and } \Pi_{n^*+1}(S_{n^*+1}) < 0$$

- **Entry thresholds:** We can define the free-entry market-structure using a step-function of the market-size supporting n firms
  - $S_1 = \frac{F_1}{[P_1 - AVC_1(q_1,W)]d(Z,P_1)}$
  - $S_2 = \frac{2F_2}{[P_2 - AVC_2(q_2,W)]d(Z,P_2)}$
  - $\ldots$
  - $S_n = \frac{nF_n}{[P_n - AVC_n(q_n,W)]d(Z,P_n)}$
- The goal of the paper is to estimate *relative* entry thresholds:

$$s_2 = \frac{S_2}{2}, s_3 = \frac{S_3}{3}, s_4 = \frac{S_4}{4}, \ldots$$

- **Economic interpretation?** If costs are independent of n, s_n is a scale-free parameter bounded below by one, and is increasing in the degree of market-power.
- **Caveat:** This ratio is not a measure of market-power. Instead it changes in s_n measures how profits *change* with competition.
  - If costs are independent of n, $s_{N+1}/s_N = 1$ implies that firms are competitive *or* fully collusive.
  - If costs are **not** independent of n, changes in the relative entry thresholds reveal either markup changes with N (i.e. conduct) or cost changes with N (e.g. barrier to entry):

$$\frac{s_{n+1}}{s_n} = \frac{F_{n+1}}{F_n} \frac{[P_n - AVC_n(q_n, W)]\, d(Z, P_n)}{[P_{n+1} - AVC_{n+1}(q_{n+1}, W)]\, d(Z, P_{n+1})} = \frac{F_{n+1}}{F_n} \frac{V_n}{V_{n+1}}$$

  - Therefore, we can make inference about market conduct (i.e. change in V_m) only under some assumptions about F_n.

Example: Free entry in the Cournot Model

SUCCESSIVE ENTRY THRESHOLD RATIOS FOR A COURNOT OLIGOPOLY MODEL WITH LINEAR DEMAND AND CONSTANT MARGINAL COSTS

	$k = 0$, MES $= \infty$		$k = 2$, MES $= 1.58$		$k = 10$, MES $= .71$	
NUMBER OF FIRMS	s_{N+1}/s_N (1)	$P_N - \mathrm{MC}_N$ (2)	s_{N+1}/s_N (3)	$P_N - \mathrm{MC}_N$ (4)	s_{N+1}/s_N (5)	$P_N - \mathrm{MC}_N$ (6)
1	2.25	7.5	2.17	6.3	2.01	.8
2	1.78	5.0	1.64	3.8	1.52	.4
3	1.56	3.8	1.42	2.7	1.34	.3
4	1.44	3.0	1.31	2.1	1.25	.2
5	1.36	2.5	1.24	1.7	1.20	.2
20	1.10	.8	1.06	.4	1.05	.0
∞	1.00	.0	1.00	.0	1.00	.0

NOTE.—Price minus marginal cost equals $15 - (Q/S) - 2kq$. Fixed costs equal five. MES denotes minimum efficient scale

Empirical Model:

- Functional form:

$$\Pi_n = S(Y, \lambda)V_n(Z, W, \alpha, \beta) - F_n(W, \gamma) + \epsilon$$

  Where,

  - *Market size:* $S(Y, \gamma) =$ town population + γ_1 nearby pop. + γ_2 pos. growth + γ_3 neg. growth + γ_4 out-of-county commuters
  - *Variable profits:* $V_N = \alpha_1 + X\beta - \sum_{n=2}^{N} \alpha_n$
  - *Fixed-cost:* $F_N = \gamma_0 + \gamma_L$ land cost + $\sum_{n=2}^{N} \gamma_n$
  - *Residual:* $\epsilon \sim N(0, 1)$.

- **Likelihood function:** Ordered-probit

$$L(N, X; \theta) = \frac{1}{M} \sum_i \log \Pr(N_i | X_i, \theta)$$

  Where,

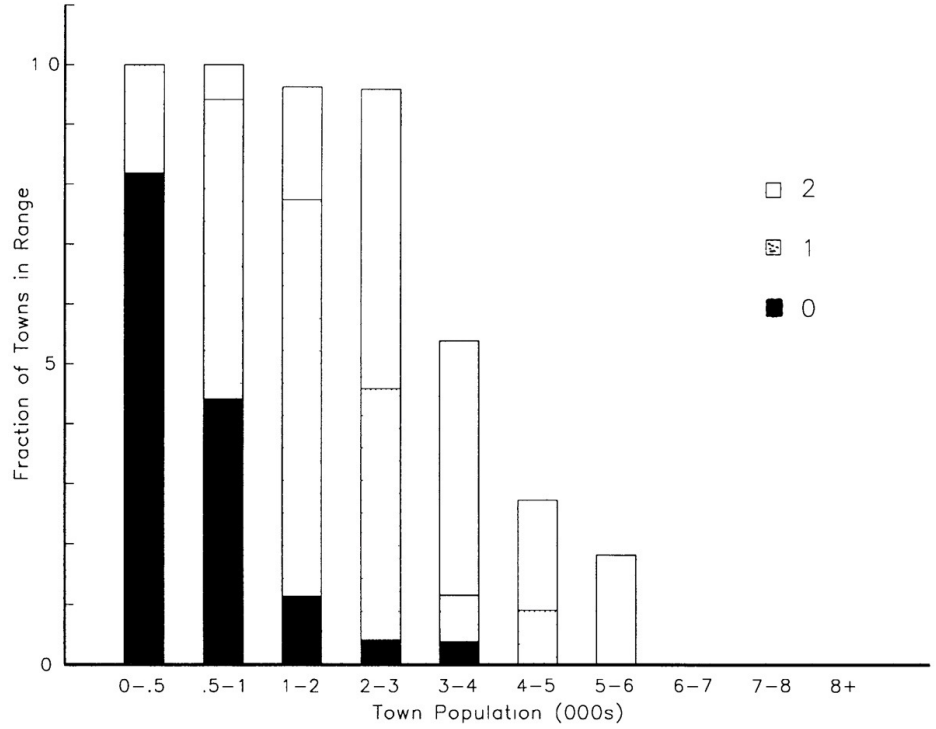
$$\Pr(N_i | X_i, \theta) = \Phi(\bar{\Pi}_{N_i}(X; \theta)) - \Phi(\bar{\Pi}_{N_i+1}(X_i; \theta))$$

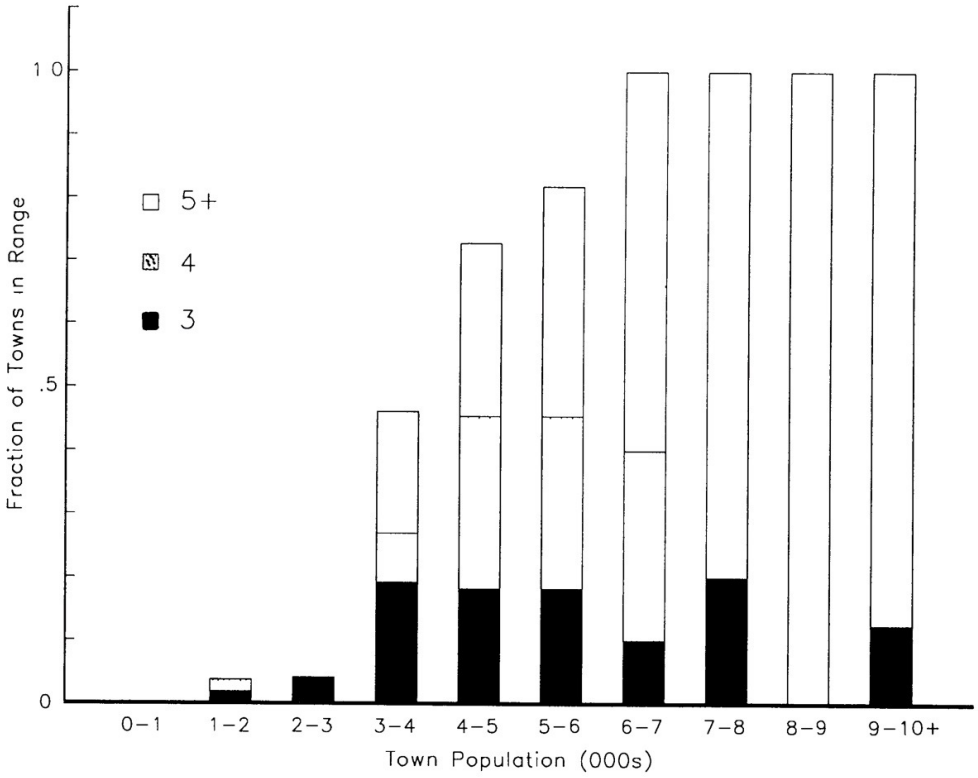
$$\Pr(0 | X_i, \theta) = 1 - \Phi(\bar{\Pi}_1(X_i; \theta))$$

- **Data:**

  - *Sample:* 202 isolated counties mostly in the western part of the US.
  - *Key variables:* Number of active retail or professional providers, demographic characteristics, growth

Graphical evidence: Decreasing entry thresholds





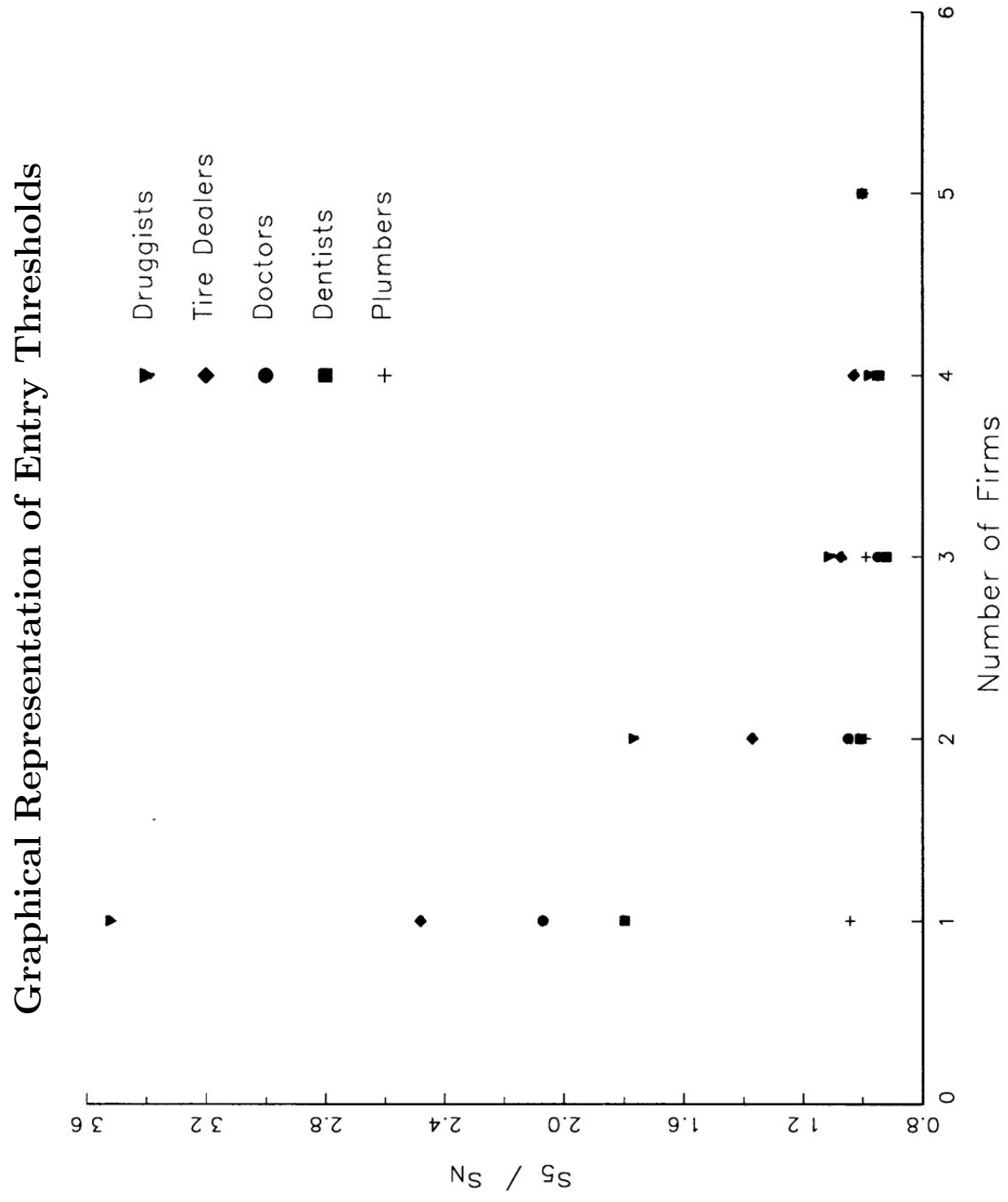
Estimation Results

Profession	Entry Thresholds (000's)					Per Firm Entry Threshold Ratios			
	S_1	S_2	S_3	S_4	S_5	s_2/s_1	s_3/s_2	s_4/s_3	s_5/s_4
Doctors	.88	3.49	5.78	7.72	9.14	1.98	1.10	1.00	.95
Dentists	.71	2.54	4.18	5.43	6.41	1.78	.79	.97	.94
Druggists	.53	2.12	5.04	7.67	9.39	1.99	1.58	1.14	.98
Plumbers	1.43	3.02	4.53	6.20	7.47	1.06	1.00	1.02	.96
Tire dealers	.49	1.78	3.41	4.74	6.10	1.81	1.28	1.04	1.03

B. Likelihood Ratio Tests for Threshold Proportionality

Profession	Test for $s_4 = s_5$	Test for $s_3 = s_4 = s_5$	Test for $s_2 = s_3 = s_4 = s_5$	Test for $s_1 = s_2 = s_3 = s_4 = s_5$
Doctors	1.12 (1)	6.20 (3)	8.33 (4)	45.06* (6)
Dentists	1.59 (1)	12.30* (2)	19.13* (4)	36.67* (5)
Druggists	.43 (2)	7.13 (4)	65.28* (6)	113.92* (8)
Plumbers	1.99 (2)	4.01 (4)	12.07 (6)	15.62* (7)
Tire dealers	3.59 (2)	4.24 (3)	14.52* (5)	20.89* (7)

Note.—Estimates are based on the coefficient estimates in table 4. Numbers in parentheses in pt. B are degrees of freedom.
* Significant at the 5 percent level.


Graphical Representation of Entry Thresholds
Druggists
Tire Dealers
Doctors
Dentists
Plumbers
S_5 / S_N
3.6
3.2
2.8
2.4
2.0
1.6
1.2
0.8
0
1
2
3
4
5
6
Number of Firms

Application 2: Entry into airline markets (Berry 1992)

Motivation:

- Airline's scale of operation (or "airport presence") at the end-point cities that constitute the city pair may give it significant advantages over other airlines.
- Does it represent an important barrier to entry? If so, are airline markets competitive?
- Identification problems:
  - The number of of competitors in a given route is endogenously determined.
  - Need a model of market structure, to understand the "causal" effect of competition on firms profits.
- Challenges:
  - Route choice is high-dimensional network choice problem (i.e. hub-spoke). Therefore, the model will focus on the "partial" equilibrium of the market structure of individual routes (taking as given the outcomes in other routes)
  - Even restricting our attention to individual routes, the market structure is complicated: (i) large number of potential entrains, (ii) firms are heterogenous (that's the whole point), and (iii) game exhibit multiple equilibria.

Model and assumptions:

- Individual profit function in market i for firm K:

$$\pi_{ik}(s) = v_i(N(s)) + \phi_{ik}$$

  where s is a $K \times 1$ vector of entry strategies, and $N(s)$ is the number of entrants.

- **Assumption:** ϕ_{ik} is common knowledge.

- Nash equilibrium conditions:

$$s^*_{ik}\pi_{ik}(s^*) \geq 0 \text{ and } (1 - s^*_{ik})\pi_{ik}(s^*) < 0, \text{for all } k = 1, 2, ..., K.$$

- Implications:

  1. In any pure-strategy Nash equilibrium, the number of entering firms is **uniquely** determined
  2. The identity of entrants is **not** uniquely determined.

- Equilibrium selection rules:

  1. Order of play is determined by idiosyncratic profits: $\phi_{i1} > \phi_{i2} > \phi_{i3} > ... > \phi_{iK}$
  2. Order of play is determined by incumbency status (i.e. companies that were active in January 1980 move first), and profits (i.e. within incumbency status, more profitable firms move first).

- Functional form:

  – Common variable profit term:

$$v_i(N) = X_i\beta + \beta \log N + \rho e_i$$

– Idiosyncratic profits:

$$\phi_{ik} = Z_{ik}\alpha + \sigma u_{ik}$$

– Scale restriction:

$$\varepsilon_{ik} = \sqrt{1-\rho^2}u_{ik} + \rho e_i$$

where e_i and u_{ik} are IID standard normal random variables.

– This lead to the reduced-form profit function:

$$\pi_{ik}(s) = X_i\beta + \beta \log N(s) + Z_{ik}\alpha + \varepsilon_{ik}$$

- Solution steps (i.e. assuming order of play):
  - Sort firms according to vector $(\phi_{i1}, \phi_{i2}, ..., \phi_{iK})$
  - Find n^* such that:

$$v_i(n^*) + \phi_{i,n^*} \geq 0$$

$$v_i(n^*+1) + \phi_{i,n^*+1} < 0$$

Simulated method of moments:

- Instead of calculating the probability of the observed strategies $\{s_{ik}\}$, we can simulate S realizations of the random profits shocks, and compute moments characterizing the equilibrium market structure.
- Let $\hat{m}(x)$ denote a vector of empirical moments describing the structure of individual markets (e.g. average number of firms, covariance between number of firms and airport presence, market size, etc)
- Simulation procedure:
  - Sample random shocks t: $(\varepsilon_{i1}^t, \varepsilon_{i2}^t, ..., \varepsilon_{iK}^t)$ for every markets i.
  - Solve the equilibrium market structure: $s^*(t)$
  - Calculate moments: $m^t(x|\theta)$
  - Moment conditions:

$$g(x, \theta) = \sum_t \hat{m}(x) - m^t(x|\theta)$$

  - Simulated GMM objective function:

$$\min_\theta g(x, \theta)^T \Omega^{-1} g(x, \theta)$$

Results from restricted models

TABLE VI

MAXIMUM LIKELIHOOD RESULTS[a]

Variable	No Heterogeneity	Only Observed Heterogeneity	No Correlation
Constant	1.00	−0.973	−1.54
	(0.056)	(0.485)	(0.815)
Population	4.33	4.16	4.32
	(0.102)	(0.180)	(0.059)
Dist	−0.184	−0.841	−0.903
	(0.034)	(0.070)	(0.112)
City2	—	1.68	1.43
		(0.479)	(0.524)
City share	—	1.20	−2.94
		(0.118)	(0.070)
δ	1.81	1.66	0.252
	(0.050)	(0.470)	(1.92)
−2 log-likelihood:	3715	3619	1732

[a]Observations are 1219 markets. Standard errors are in parentheses.

Results full restricted model: Two selection assumptions

TABLE VII

Simulation Estimates[a]

Variable	Most Profitable Move First	Incumbents Move First
Constant	−5.32	−3.20
	(0.354)	(0.258)
Population	1.36	5.28
	(0.239)	(0.343)
Dist	1.72	−1.45
	(0.265)	(0.401)
City2	4.89	5.91
	(0.295)	(0.149)
City Share	4.73	5.41
	(0.449)	(0.206)
δ	0.527	4.90
	(0.119)	(0.206)
ρ	0.802	0.050
	(0.105)	(0.048)
Value of the objective fn:	33.3	26.2

[a]Observations are 1219 markets. Standard errors are in parentheses.

Application 3: Market Structure and Multiple Equilibria (Ciliberto and Tamer 2009)

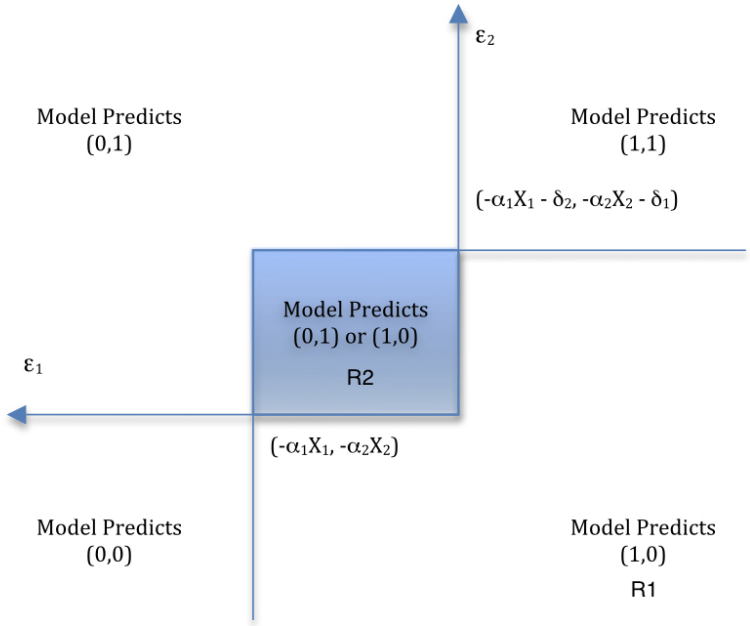
- **Motivation:** A major criticism of the previous two models is the "ad-hoc" restrictions imposed to select a unique equilibrium.
  - Bresnahan and Reiss (1991): No firm heterogeneity
  - Berry (1992): Sequential move
  - In addition, both approaches ex-ante restrict the sign of the competitive effect (rule out complementaries)
- Ciliberto and Tamer (2009) propose a different approach based on an *incomplete* model. Why is important?
  - In their application, heterogeneity across firms is first-order
  - Major carriers use *hub-spoke* network to dominate routes, and *low-cost* carriers compete on most major direct routes

- **Illustration:** Bresnahan and Reiss (1990)

$$y_{1m} = 1\left[X_{1m}\alpha_1 + \delta_2 y_{2m} + \epsilon_{1m} \geq 0\right]$$
$$y_{2m} = 1\left[X_{2m}\alpha_2 + \delta_1 y_{1m} + \epsilon_{2m} \geq 0\right]$$

  where $(\epsilon_{1m}, \epsilon_{2m})' \sim N(0, \Sigma)$.

- Indeterminacy in a figure:



- Likelihood contributions:

$$\begin{aligned}
\Pr((1,1)|X,\theta) &= \Pr(\epsilon_{i1} \geq -X_{1m}\alpha_1 - \delta_2; \epsilon_{i2} \geq -X_{2m}\alpha_2 - \delta_1) \\
\Pr((0,0)|X,\theta) &= \Pr(\epsilon_{i1} \leq -X_{1m}\alpha_1; \epsilon_{i2} \leq -X_{2m}\alpha_2) \\
\Pr((1,0)|X,\theta) &= \Pr((\epsilon_{i1}, \epsilon_{i2}) \in R_1(X,\theta)) \\
&\quad + \int \underbrace{\Pr((1,0)|\epsilon_{i1}, \epsilon_{i2}, X, \theta)}_{\text{Selection probability}} 1((\epsilon_{i1}, \epsilon_{i2}) \in R_2(X,\theta)) dF_{\epsilon_{i1},\epsilon_{i2}}
\end{aligned}$$

- **Problem:** The selection probability is an infinite dimensional nuisance parameter.

- Model restrictions:

$$\begin{aligned}\Pr((\epsilon_{i1},\epsilon_{i2}) \in R_1(X,\theta)) &\leq \Pr((1,0)|X,\theta) \\ &\leq \Pr((\epsilon_{i1},\epsilon_{i2}) \in R_1(X,\theta)) + \Pr((\epsilon_{i1},\epsilon_{i2}) \in R_2(X,\theta))\end{aligned}$$

$$H_1(X,\theta) \leq \Pr((1,0)|X,\theta) \leq H_2(X,\theta)$$

- **Note:** If the equilibrium is unique for X and θ, the model implies a unique likelihood: $H_1 = H_2$.
- This *robust* prediction of the model leads naturally to an estimator based on *conditional moment inequality* restrictions.
- **Notation:**
  - K = Number of players
  - $\{\mathbf{y}_1, \ldots, \mathbf{y}_{2^K}\} = 2^K \times K$ matrix of possible market-structure outcomes
  - $\Pr(X)$ is $2^K \times 1$ vector with element j, $\Pr(\mathbf{y}_j|X)$, the *observed* probability of observing outcome $\mathbf{y}'$ in markets with characteristics X.
  - $X \in \mathcal{X}$ denotes the *finite* set of market characteristics, and $J = |\mathcal{X}|$ denotes the number of elements in that set.
- **Profits:** Let $X_m = \{Z_{im}, W_{im}, S_m\}_{i=1,\ldots,K}$

$$\pi_{im}(\mathbf{y}, X_m, \epsilon) = S_m\alpha_i + Z_{im}\beta_i + W_{im}\gamma_i + \sum_{k\neq i} y_k\delta_{i,j} + \sum_{k\neq i} y_k Z_{im}\phi_{ij} + \epsilon_{im}$$

  where ϵ_{im} is the sum of four normal errors: u_m, u_m^o, u_m^d, u_{im}.
- **Nash Equilibrium:** Market structure $\mathbf{y}$ is a Nash equilibrium of the simultaneous entry game in market (X, ϵ) if

$$y_i \times \left(S_m\alpha_i + Z_{im}\beta_i + W_{im}\gamma_i + \sum_{k\neq i} y_k\delta_{i,k} + \sum_{k\neq i} y_k Z_{im}\phi_{ik} + \epsilon_{im} \right) \geq 0, \forall i = 1, \ldots, K$$

- **Conditional moments:** At the true underlying parameter vector θ^0, the expeted market-structure $E(\mathbf{y}|X) = \Pr(\mathbf{y}|X)$ satisfies the following inequality:

$$H_1(\mathbf{y}|X, \theta^0) \leq \Pr(\mathbf{y}|X) \leq H_2(\mathbf{y}|X, \theta^0), \quad \forall X \in \mathcal{X}$$

- In matrix form, the following inequality vector holds:

$$\mathbf{H}_1(X, \theta^0) \leq \Pr(X) \leq \mathbf{H}_2(X, \theta^0), \quad \forall X \in \mathcal{X}.$$

  This gives us 2^K moment inequalities.

- **Identification:** In theory the model is *point-identified* if (a) X contains at least one exclusion restriction, and (b) this characteristics varies enough across firms/markets (i.e. large support assumption).
  - The logic of the large-support assumption is that the problem boils down to a single-agent entry game if $x_{im} \to \infty$ for some (i, m).
  - In practice, it is infeasible to verify that this condition holds, and so the inference methodology needs to be robust to partial identification.

- **Estimation:**
  - Objective function:

$$Q(\theta) = \sum_{j=1}^{J} w_j \left[||(P(X_j) - H_1(X_j, \theta))_-|| + ||(P(X_j) - H_2(X_j, \theta))_+|| \right]$$

    where ω_j is the density of markets with characteristics X_j,
    $(A)_- = [a_1 1(a_1 \leq 0), \ldots, a_{2^k} 1(a_{2^k} \leq 0)]$,
    $(A)_+ = [a_1 1(a_1 \geq 0), \ldots, a_{2^k} 1(a_{2^k} \geq 0)]$, and $||B|| = \sum_{i=1}^{w^K} B_i^2$.

- Estimation procedure:
  - Discretize X in J bins and estimate sample frequency $\hat{w}_j$
  - Estimate the empirical analogue of $\hat{P}(X_j)$ for all j
  - Use simulation methods to evaluate the objective function $Q_n(\theta)$. How?
  - Sample $(\epsilon_1, \ldots, \epsilon_K) \sim N(0, \Sigma)$
  - Solve the model for each (X, ϵ^i):
    1. Set $H_1(\mathbf{y}|X, \epsilon^i, \theta) = 0$ and $H_2(\mathbf{y}|X, \epsilon^i, \theta) = 0$ for all $\mathbf{y}$.
    2. Loop over possible market-structures: $\{\mathbf{y}_1, \ldots, \mathbf{y}_{2^K}\}$
    3. If the NE conditions are satisfied for $\mathbf{y}$, set $H_2(\mathbf{y}|X, \epsilon^i, \theta) = H_2(\mathbf{y}|X, \epsilon^i, \theta) + 1$
    4. After investigating all possible market-structures, if there is only one NE configuration then set $H_1(\mathbf{y}|X, \epsilon^i, \theta) = 1$
  - Calculate the average across simulation draws for every 2^K configurations:

$$\hat{H}_2(\mathbf{y}|X, \theta) = \frac{1}{I} \sum_i H_2(\mathbf{y}|X, \epsilon^i, \theta)$$

$$\hat{H}_1(\mathbf{y}|X, \theta) = \frac{1}{I} \sum_i H_1(\mathbf{y}|X, \epsilon^i, \theta)$$

    This gives us a measure of the upper and lower bounds on the probability that $\mathbf{y}$ is an equilibrium in markets X.
  - Empirical analogue of the objective function:

$$Q_n(\theta) = \sum_{j=1}^{J} \hat{w}_j \left[||(\hat{P}(X_j) - \hat{H}_1(X_j, \theta))_-|| + ||(\hat{P}(X_j) - \hat{H}_2(X_j, \theta))_+|| \right]$$

- Inference:
  - If the conditions for point-identification could be evaluated, we could proceed with estimation/inference as usual by minimizing (a smoothed version) of the objective function.
  - Since this cannot be done in this case, Ciliberto and Tamer (2009) conduct inference by bootstrapping the test-statistic following the method proposed by Chernozhukov, Hong, and Tamer (2007)
- Data and market definition:
  - *Source:* 2001 Airline Origin and Destination Survey (DB1B).
  - *Market:* Trip between two airports associated with the top 100 MSAs, irrespective of intermediate transfer points and of the direction of the flight.
  - *Players:* American, Delta, United, Southwest, "medium airlines", and "low-cost carriers".
  - *Key variables:*
    * Airport presence: The average of the carrier's routes shares at the two endpoints.
    * Cost: Sum of distance between endpoints and the carrier's nearest hub/ Direct distance
    * Wright Amendment: Indicator variable equal to one for markets that have restricted connections to Texas' Love Field airport.
    * Control variable: Market size, income, growth, non-stop distance, distance to nearest alternative airport.

Estimation Results

	Berry (1992)	Heterogeneous Interaction	Heterogeneous Control	Firm-to-Firm Interaction
Competitive fixed effect	[−14.151, −10.581]			
AA		[−10.914, −8.822]	[−9.510, −8.460]	
DL		[−10.037, −8.631]	[−9.138, −8.279]	
UA		[−10.101, −4.938]	[−9.951, −5.285]	
MA		[−11.489, −9.414]	[−9.539, −8.713]	
LCC		[−19.623, −14.578]	[−19.385, −13.833]	
WN		[−12.912, −10.969]	[−10.751, −9.29]	
LAR on LAR				
LAR: AA, DL, UA, MA				[−9.086, −8.389]
LAR on LCC				[−20.929, −14.321]
LAR on WN				[−10.294, −9.025]
LCC on LAR				[−22.842, −9.547]
WN on LAR				[−9.093, −7.887]
LCC on WN				[−13.738, −7.848]
WN on LCC				[−15.950, −11.608]
Airport presence	[3.052, 5.087]	[11.262, 14.296]	[10.925, 12.541]	[9.215, 10.436]
Cost	[−0.714, 0.024]	[−1.197, −0.333]	[−1.036, −0.373]	[−1.060, −0.508]
Wright	[−20.526, −8.612]	[−14.738, −12.556]	[−12.211, −10.503]	[−12.092, −10.602]
Dallas	[−6.890, −1.087]	[−1.186, 0.421]	[−1.014, 0.324]	[−0.975, 0.224]
Market size	[0.972, 2.247]	[0.532, 1.245]	[0.372, 0.960]	[0.044, 0.310]
WN			[0.358, 0.958]	
LCC			[0.215, 1.509]	

Takeaways:

- Heterogenous competitive effects are economically and statistically significant.
- With heterogenous effects, multiple equilibria is empirically relevant. 53% of markets exhibit equilibria with different **number** of firms.

Variable effects: Competition × Airport presence

	Independent Unobs	Variance–Covariance	Only Costs
Fixed effect			
AA	[−9.433, −8.485]	[−8.817, −8.212]	[−11.351, −9.686]
DL	[−10.216, −9.255]	[−9.056, −8.643]	[−12.472, −11.085]
UA	[−6.349, −3.723]	[−4.580, −3.813]	[−10.671, −8.386]
MA	[−9.998, −8.770]	[−7.476, −6.922]	[−11.906, −10.423]
LCC	[−28.911, −20.255]	[−14.952, −14.232]	[−11.466, −8.917]
WN	[−9.351, −7.876]	[−6.570, −5.970]	[−12.484, −10.614]
Variable effect			
AA	[−5.792, −4.545]	[−4.675, −3.854]	
DL	[−3.812, −2.757]	[−3.628, −3.030]	
UA	[−10.726, −5.645]	[−8.219, −7.932]	
MA	[−6.861, −4.898]	[−7.639, −6.557]	
LCC	[−9.214, 13.344]		
WN	[−10.319, −8.256]	[−11.345, −10.566]	
Airport presence	[14.578, 16.145]	[10.665, 11.260]	
Cost	[−1.249, −0.501]	[−0.387, −0.119]	
AA			[−0.791, 0.024]
DL			[−1.236, 0.069]
UA			[−1.396, −0.117]
MA			[−1.712, 0.072]
LCC			[−17.786, 1.045]
WN			[−0.802, 0.169]
Wright	[−17.800, −16.346]	[−16.781, −15.357]	[−14.284, −10.479]
Dallas	[0.368, 1.323]	[0.839, 1.132]	[−5.517, −2.095]
Market size	[0.230, 0.535]	[0.953, 1.159]	[1.946, 2.435]
WN	[0.260, 0.612]	[0.823, 1.068]	
LCC	[−0.432, 0.507]		
Market distance	[0.009, 0.645]	[0.316, 0.724]	[−0.039, 1.406]
WN	[−3.091, −1.819]	[−2.036, −1.395]	
LCC	[−1.363, 1.926]		
Close airport	[−0.373, 0.422]	[0.400, 1.433]	[3.224, 6.717]
WN	[1.164, 3.387]	[2.078, 2.450]	
LCC	[1.059, 3.108]	[1.875, 2.243]	
U.S. center distance	[−9.271, 0.506]	[0.015, 0.696]	[2.346, 3.339]
WN	[0.276, 1.008]	[0.668, 1.097]	
LCC	[−0.930, 0.367]		
Per capita income	[0.929, 1.287]	[0.824, 1.052]	[1.416, 2.307]
Income growth rate	[0.136, 0.331]	[0.151, 0.316]	[1.435, 2.092]
Constant	[−0.522, 0.163]	[−0.827, −0.523]	[−12.404, −10.116]
MA_m	[0.664, 1.448]	[0.279, 0.747]	
LCC	[−1.528, −0.180]	[−0.233, 0.454]	
WN	[1.405, 2.215]	[1.401, 1.659]	
Function value	1616	1575	1679
Multiple in identity	0.9538	0.9223	0.9606
Multiple in number	0.6527	0.3473	0.0728
Correctly predicted	0.3461	0.3375	0.3011

Application 4: Entry into spatially differentiated markets (Seim 2006)

Motivation:

- Previous literature has focused on entry into **homogenous** good markets.
- Important question: What determines the variety of products offered?
- Horizontal differentiation hypothesis: The profits are declining faster in the number of "similar" products, compare to the number of "differentiated" products.
- In principle, we can answer this question by estimating a demand and pricing mode of competition. However, prices and quantity data are not readily available for all industries, and standard static oligopoly models rely on the assumption that the location of products in exogenous.
- Empirical challenges:
  - Common information entry/location-choice games are complicated to solve numerically, and exhibit multiple equilibria.
  - Existence of pure-strategy Nash equilibrium is not guaranteed.
  - Solution: Introduce incomplete information
- Application: Entry and location choice in the video-store industry (i.e. cassettes and DVDs rental)

Model description:

- Location choices: $l_i \in \{0, 1, ..., L\}$
- Profit function:

$$\Pi_{im}(l, l_{-i}) = X_{lm}\beta + \sum_{b=1}^{B} \delta_b n_b(l, l_{-i}) + \xi_m + \varepsilon_{ilm}$$

  where $n_b(l_i, l_{-i})$ denotes the number of competitors located with distance κ_b and κ_{b+1} of location l_i, and ε_{ilm} is an extreme-value shock (private information).

- Let $\tilde{p}_{lm}$ denote the belief that firms have about the probability that one firm enter location l in market m.
- Then the expected profit of entering location l is given by:

$$\pi_{im}(l, \tilde{p}_m) = X_{lm}\beta + \sum_{b=1}^{B} \delta_b E(n_b(l, l_{-i})|\tilde{p}_m) + \xi_m + \varepsilon_{ilm}$$

$$\pi_{im}(l, \tilde{p}_m) = X_{lm}\beta + \sum_{b=1}^{B} \delta_b \tilde{n}(l) + \xi_m + \varepsilon_{ilm}$$

  where $\tilde{n}_b(l) = \sum_{l'} 1(\kappa_b < D(l, l') < \kappa_{b+1})\tilde{p}_{l'm}N_m$, and N_m is the number of entrants in market m.

- Under this belief structure, the best-response probability mapping for location l is:

$$p_{lm} = \Psi_{lm}(\tilde{p}_m) = \frac{\exp(X_{lm}\beta + \sum_{b=1}^{B} \delta_b \tilde{n}_b(l) + \xi_m)}{\sum_{l'} \exp(X_{l'm}\beta + \sum_{b=1}^{B} \delta_b \tilde{n}_b(l) + \xi_m)}$$

$$p_{lm} = \Psi_{lm}(\tilde{p}_m) = \frac{\exp(X_{lm}\beta + \sum_{b=1}^{B} \delta_b \tilde{n}_b(l))}{\sum_{l'} \exp(X_{l'm}\beta + \sum_{b=1}^{B} \delta_b \tilde{n}_b(l))}$$

- A symmetric perfect-bayesian Nash equilibrium is a fixed point of this mapping:

$$p^*_{lm} = \Psi_{lm}(p^*_m), \text{ for all } l.$$

- Existence? Guaranteed following Browser fixed-point theorem
- Uniqueness?
  - When $B = 2$ and $\delta_1 < \delta_2 < 0$: There exist a unique vector of location choice-probabilities that solve the best-response mapping.
  - Simulations show that this result extends for $B = 3$, but it is not guaranteed.

Estimation:

- Data: Cross-section of markets (i.e. isolated groups of cities) with the location of every video rental stores in 1999.
- Location: Census-tract
- Potential number of entrants: 50 or twice the actual number of entrants
- Likelihood function:
  - Choice probabilities are implicitly defined by the equilibrium conditions: $p^*_{lm} = \Psi_{lm}(p^*_m),$ for all l and m.
  - Market-level random effect: $\xi_m \sim N(0, \sigma^2_\xi)$
  - Notice that ξ_m does not enter the choice-probabilities, but it affects the entry probability:

$$\Pr(\text{entry}) = \frac{\exp(\xi)\left[\sum_l \exp(\bar{\pi}_l(p^*))\right]}{1 + \exp(\xi)\left[\sum_l \exp(\bar{\pi}_l(p^*))\right]}$$

    and $N_m = \bar{N} \times \Pr(\text{entry})$
  - Therefore, the market-level error term can be inverted from the data:

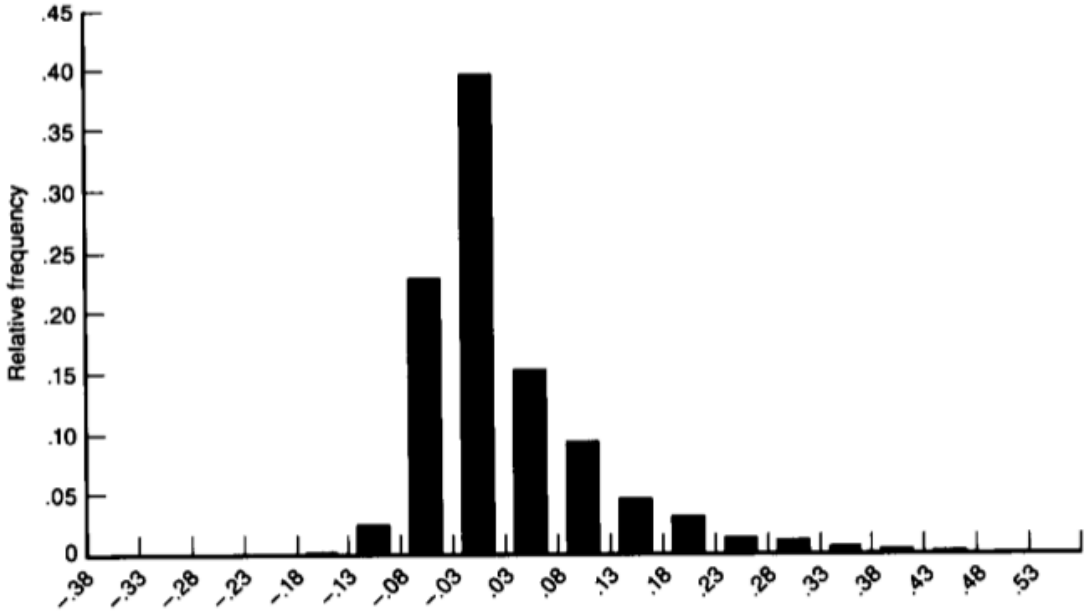
$$\xi_m = \log N_m - \log(\bar{N} - N_m) - \log\left[\sum_l \exp(\bar{\pi}_l(p^*))\right]$$

  - This produces the following joint likelihood contribution for market m

$$l_m = \sum_l n_{lm} \log p^*_{lm} + \phi(\xi_m(p^*)/\sigma_\xi)$$

FIGURE 3

DISTRIBUTION OF PREDICTION ERRORS, LOCATION-CHOICE PROBABILITIES



	Potential Entrant Pool			
	2 × Total Entrants		50 Firms	
Variable	Coefficient (Standard Error)	Marginal Effect	Coefficient (Standard Error)	Marginal Effect
Population_0 (000)	1.8191	.0333	2.1258	.0393
	(.1534)		(.1764)	
Population_1 (000)	1.3109	.0236	1.7349	.0314
	(.1200)		(.1498)	
Population_2 (000)	.6070	.0121	1.1348	.0227
	(.1192)		(.1486)	
Business Density	−.8077	−.0155	−.8889	−.0173
	(.1458)		(.1477)	
Average Per Capita Income_0 (0000)	.9309	.0180	1.0380	.0204
	(.1136)		(.1233)	
Average Per Capita Income_1 (0000)	1.0081	.0193	.9188	.0178
	(.2081)		(.2043)	
Average Per Capita Income_2 (0000)	.4851	.0092	.4884	.0094
	(.2512)		(.2601)	
γ_0	−3.4520		−3.3853	
	(.3111)		(.3266)	
γ_1	−1.0103		−1.0087	
	(.0745)		(.0923)	
γ_2	−.3448		−.4870	
	(.0738)		(.0934)	
σ	3.5829		4.6760	
	(.3110)		(.4316)	
μ	−2.8764		−7.0364	
	(1.3425)		(1.5801)	

Note: Results based on 1999 demographic and firm data. Subscript 0 denotes the immediately adjacent locations to the chosen tract, within .5 miles in distance; subscript 1 denotes tracts at .5 to 3 miles in distance from the chosen tract; and subscript 2 denotes tracts at more than 3 miles distance from the chosen tract. Tract-level business density is defined as the number of establishments (0000) per square mile. γ denotes competitive effects, and σ and μ are the estimates of the parameters of the distribution of ξ.

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