Search and Matching Frictions

Jean-François Houde
Cornell University & NBER

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Introduction

- In IO, search models are used mostly to explain:
  - Deviations from the *law of one price*: Existence of price dispersion for (nearly) homogenous goods (Stigler (1961))
  - *Misallocation* of products: Why consumers choose inferior products
- The presence of search frictions is a source of market power (like differentiation)
  - **Diamond paradox:** If every consumer must incur search cost $\epsilon > 0$ to learn the price of other goods, the unique Nash equilibrium is the monopoly price.

Introduction

- Much of the early theory literature has focussed on deriving game theory models that produce **equilibrium price dispersion**.
- Two important frameworks:
  - Varian (1980)/Stahl (1989): Temporal dispersion and sequential search
  - Burdett and Judd (1983): Price dispersion and simultaneous search
- Both models generate dispersion by incorporating a positive mass of fully informed consumers in the market, and allowing for mixed-strategy equilibrium.
  - The Burdett and Judd model is the workhorse model in macro-labor (see Burdett and Mortensen (1998))

Varian (1980): "A Model of Sales"

- Notation:
  - r: Reservation price of consumers (common)
  - I: Mass of informed consumers
  - M: Mass of uninformed consumers
  - n: Number of stores (endogenous)
  - $C(q) = F + cq$: Cost function (common)
- Timing:
  1. **Entry:** n such that $E(\pi|n) = 0$
  2. **Pricing:** Density function $f(p)$
  3. **Demand:** (i) Informed buyers choose from the lowest price store, and (ii) uninformed buyers are randomly assigned to each store (i.e. $U = M/n$)
- **Note:** With free entry, U is an endogenous outcome of the model (i.e. contact rate)

Part 1: Properties the Equilibrium Price Distribution

1. Range of the distribution: (p^*, r)
   - Maximum: Reservation price (r)
   - Minimum: Lowest average cost when serving the entire market

$$p^* = \frac{F + c}{I + U}$$

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2. Existence of price dispersion:
   - Suppose all stores where posting p: $r \geq p > p^*$ (i.e. positive profits)
   - Undercutting by ϵ is a profitable deviation: Capture informed consumers.
   - If $p = p^*$ (zero profits): Increasing prices by ϵ allow firms to catch $1/n$ uninformed consumers
   - Therefore, if $n - 1$ firms post a single price, firm i always has a profitable deviation.

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3. No mass point in $f^*(p)$:
   - Suppose again $f(\cdot)$ has a mass at price p
   - This implies that firms can tie at the minimum with positive probability (i.e. split I)
   - Profitable deviation: Lower price by ϵ to eliminate the probability of ties

Part 2: Mixed-Strategy Equilibrium

- Given n, the three properties above implies that firms will play a mixed-strategy by randomizing prices over the range $[p^*, r]$.
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- **Expected profit:** Two components
  - Profits from selling to informed buyers: $\pi_s(p)(1 - F(p))^{n-1}$
  - Profits from selling to uninformed buyers: $\pi_f(p)\left[1 - (1 - F(p))^{n-1}\right]$
  - Where $\pi_s(p) = (p - c)(U + I) - F$ (i.e. sales) and $\pi_f(p) = (p - c)U - F$ (i.e. full price).

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  - Where $\pi_s(p) = (p - c)(U + I) - F$ (i.e. sales) and $\pi_f(p) = (p - c)U - F$ (i.e. full price).
- For a fixed number of stores, mixed-strategy equilibrium condition implies that firms are indifferent between posting any price in the support:

$$\pi_s(p)(1 - F(p))^{n-1} + \pi_f(p)\left[1 - (1 - F(p))^{n-1}\right]$$

$$= \pi_s(p')(1 - F(p'))^{n-1} + \pi_f(p')\left[1 - (1 - F(p'))^{n-1}\right] \quad \forall p', p \in [p^*, r]$$

Part 3: Solution Method

- **Note:** By definition $F(r) = 1$.
- Therefore, comparing p and r gives us:

$$\pi_s(p)(1 - F(p))^{n-1} + \pi_f(p)\left[1 - (1 - F(p))^{n-1}\right]$$

$$= \pi_s(r)(1 - F(r))^{n-1} + \pi_f(r)\left[1 - (1 - F(r))^{n-1}\right] = \pi_f(r)$$

$$\Rightarrow \frac{\pi_f(r) - \pi_f(p)}{\pi_s(p) - \pi_f(p)} = (1 - F(p))^{n-1}$$

$$\Rightarrow F(p) = 1 - \left(\frac{\pi_f(r) - \pi_f(p)}{\pi_s(p) - \pi_f(p)}\right)^{1/n-1}$$

- The density function is obtained by differentiating $F(p)$ wrt p

Part 4: Free Entry and Lower Bound

- With free entry, expected profit is **zero**
- **Example:** $C(q) = F$ and $MC = 0$
  - Zero profit condition: $\pi_f(r) = r\frac{M}{n} - F = 0 \rightarrow n^* = rM/F$
  - Endogenous contact rate of the uninformed: $U^* = M/n = F/r$
  - Lower bound: $\pi_s(p^*) = p^*(I + U^*) - F = 0 \rightarrow p^* = \frac{F}{I+F/r}$
- Density of prices with free entry:

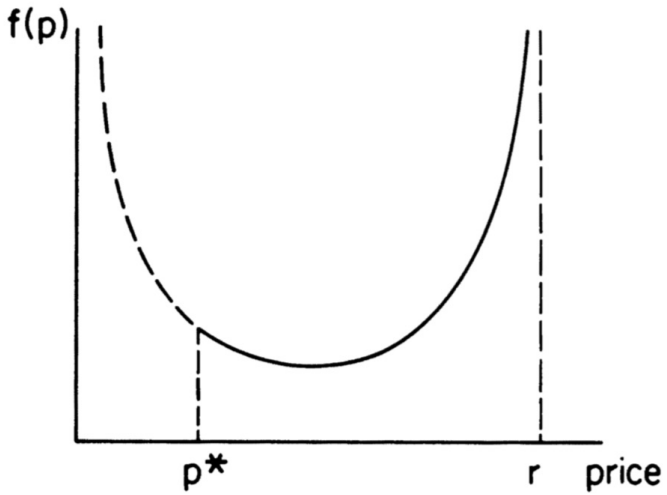


FIGURE 2. GRAPH OF $f(p)=1/p(1-p/r)$

Extensions

- Simultaneous search: Burdett and Judd (1983)
  - Uninformed consumers choose to exert effort *ex-ante*
  - Search effort = Number of price quotes (fixed)
- Sequential search: Stahl (1989)
  - Uninformed consumers endogenously decide to search using a reservation price strategy
  - If search cost is common, and initial quote is "free" the equilibrium prediction is very similar.
- Informative advertising: Robert and Stahl (1993)
  - The number of informed consumers is function of the advertising investment of firms
  - Two-stage game: (i) advertising, and (ii) pricing.
- Many others...

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  - **Reduced-form tests:** Sorensen (2000) [frequent vs infrequent purchases], Brown and Goolsbeem (2002) [online shopping], Chandra and Tappata (2011) [test for mixed-stragegy hypothesis], Allen, Clark, and Houde (2014) [distributional effects of mergers]

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  - **Equilibrium restrictions:** Hong and Shum (2006), Hortacsu and Syverson (2004), Wildenbeest (2011), Gavazza (2016), Allen, Clark, and Houde (2017), Salz (2017).

Hong and Shum (2006): Using price distributions to estimate search costs

- Notation:
  - F_p: Observed price distribution (equilibrium)
  - $F_c(\cdot)$: Search cost distribution (per store)
  - r: Common retailers' selling cost
- Assumption: There is a continuum of firms and consumers.

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  - r: Common retailers' selling cost
- Assumption: There is a continuum of firms and consumers.
- Two search protocol (given $F_p(\cdot)$):
  1. Simultaneous:

$$I(c) = \arg\min_I c(I-1) + \int_{\underline{p}}^{\bar{p}} p \underbrace{I(1-F_p(p))^{I-1} f_p(p)}_{\text{Density of the lowest price}} dp = \arg\min_I c(I-1) + Ep_{\min,I}$$

  2. Sequential:

$$c = \int_{\underline{p}}^{z(c)} (z-p) f_p(p) dp = \int_{\underline{p}}^{z(c)} F_p(p) dp \text{ [integration by part]}$$

where $z(c)$ is the *reservation price* of a consumer with search cost c.

Identification: Simultaneous Search

- Since consumers sample prices at random, consumers with lower search costs pay will lower prices on average.
  - The simultaneous search model is therefore equivalent to a *vertical differentiation* model (i.e. ordered)
  - For each number price quote n, there exists a consumer indifferent between searching n and $n+1$.
- **Observation 1:** The marginal expected savings can be computed directly from the price data $\hat{F}_p$

$$\Delta_1 = Ep_{\min,1} - Ep_{\min,2}$$

$$\Delta_2 = Ep_{\min,2} - Ep_{\min,3}$$

$$\dots$$

- **Observations 2:** Consumers with $c \in (\Delta_{n+1}, \Delta_n)$ obtain n quotes, and $\Delta_0 = +\infty$

Identification: Simultaneous Search (continued)

- Let $\tilde{q}_n$ denotes the proportion of consumers searching n prices in equilibrium.
- The expected profit of a firm charging p is given by:

$$\Pi(p) = (p - r)\left[\sum_k \tilde{q}_k (1 - F_p(p))^{k-1}\right]$$

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- Since $F_p(p)$ is obtained by firms playing a mixed-strategy equilibrium, the following indifference condition holds:

$$(\bar{p} - r)\tilde{q}_1 = (p - r)\left[\sum_k \tilde{q}_k (1 - F_p(p))^{k-1}\right], \quad \forall p \in [\underline{p}, \bar{p}]$$

- **Data:**
  - IID sample of n prices: $\hat{\underline{p}} = p_1 < p_2 < p_3 < \cdots < p_n = \hat{\bar{p}}$.
  - Consumers cannot search more than $K < n - 1$ prices (assumption)

Identification: Simultaneous Search (continued)

- The equilibrium condition implies a system of $n-1$ linear equations with K unknowns:

$$(\hat{\bar{p}} - r)\tilde{q}_1 = (p_i - r)\left[\sum_{k=1}^{K} \tilde{q}_k (1 - \hat{F}_p(p_i))^{k-1}\right], \quad \forall i = 1, \ldots, n-1$$

where $\hat{F}_p(\cdot)$ is the estimated CDF of prices.

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  where $\hat{F}_p(\cdot)$ is the estimated CDF of prices.
- From this system we can estimate: $\{r, \tilde{q}_1, \ldots, \tilde{q}_{K-1}\}$
- This allow us to estimate CDF of the search cost distribution at K percentiles:

  $\tilde{q}_k = F_c(\Delta_{k-1}) - F_c(\Delta_k) =$ Proportion of consumers searching k prices

- A parametric search cost distribution is then fitted to match the K percentiles.

Identification: Sequential Search

- Recall that $z(c)$ is the *reservation price* of consumers of type c:

$$c = \int_{\underline{p}}^{z(c)} F_p(p)dp \quad \text{[fixed-point]}$$

  - By definition, consumers will **not** buy from stores with prices above their reservation price.

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- **Note:** Since $z(c)$ can be above $\bar{p}$, some consumers will not search in equilibrium: $\bar{p}(c_i) = \min\{z(c_i), \bar{p}\}$.
- Let $G(p)$ denotes the distribution of reservation prices, $\bar{p}(c)$, among consumers
  - This distribution can be computed from $F_p(\cdot)$ and $F_c(\cdot)$.

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  - This distribution can be computed from $F_p(\cdot)$ and $F_c(\cdot)$.
- **Important:** The demand function facing a store with price p, is the fraction of consumers with reservation price **above** p.

$$D(p) = 1 - G(p)$$

Identification: Sequential Search

- Indifference condition (similar to Varian):

$$(\bar{p} - r)(1 - G(\bar{p})) = (p - r)(1 - G(p)), \quad \forall p \in [\underline{p}, \bar{p}]$$

- As before we can evaluate this condition at the observed prices: $p_1 < p_2 < \cdots < p_n$. This gives us $n - 1$ equations, with n unknowns!
- Therefore, the model cannot be estimated non-parametrically as before.
- Instead, Hong-Shum propose estimate a parametric search cost distribution $f_c(c_i|\theta)$. This distribution is identified by the following equilibrium conditions (see likelihood in paper):

$$(\hat{\bar{p}} - r)(1 - G(\hat{\bar{p}})) = (p_i - r)(1 - G(p_i)), \quad \forall i = 1, \ldots, n.$$

$$G(p_i) = F_c(z(c) < p_i|\theta) \text{ and } c = \int_{\underline{p}}^{z(c)} \hat{F}_p(p)dp$$

Summary Statistics

TABLE 1 Summary Statistics on Prices for Different Products

Product	n	List	Mean	Standard Deviation	Median	$\underline{p}$	$\overline{p}$
Stokey-Lucas	19	60.50	66.60	5.64	64.98	59.75	86.80
Lazear	17	31.95	34.73	2.48	35.27	29.51	37.70
Billingsley	20	99.95	95.48	5.87	98.90	83.58	100.87
Duffie	15	65.00	62.71	4.91	63.48	50.58	69.95

Note: Including shipping and handling costs. Price data for all products downloaded from Pricescan.com and MySimon.com: February 5, 2002. Summary price including S&H costs may not exceed the corresponding summary price without S&H costs, since we could not determine the shipping and handling charges from some of the websites.

Estimation Results: Simultaneous Search Model

TABLE 2 Search-Cost Distribution Estimates for Nonsequential-Search Model

Product	K[a]	M[b]	$\tilde{q}_1^c$	$\tilde{q}_2$	$\tilde{q}_3$	Selling Cost r	MEL Value
Parameter estimates and standard errors: nonsequential-search model							
Stokey-Lucas	3	5	.480 (.170)	.288 (.433)		49.52 (12.45)	102.62
Lazear	4	5	.364 (.926)	.351 (.660)	.135 (.692)	27.76 (8.50)	84.70
Billingsley	3	5	.633 (.944)	.309 (.310)		69.73 (68.12)	199.70
Duffie	3	5	.627 (1.248)	.314 (.195)		35.48 (96.30)	109.13

Search-cost distribution estimates

	Δ_1	$F_c(\Delta_1)$	Δ_2	$F_c(\Delta_2)$	Δ_3	$F_c(\Delta_3)$
Stokey-Lucas	2.32	.520	.68	.232		
Lazear	1.31	.636	.83	.285	.57	.150
Billingsley	2.90	.367	2.00	.058		
Duffie	2.41	.373	1.42	.059		

[a] Number of quantiles of search cost F_c that are estimated (see equation (5)). In practice, we set K and M to the largest possible values for which the parameter estimates converge. All combinations of larger K and/or larger M resulted in estimates that either did not converge or did not move from their starting values (suggesting that the parameters were badly identified).

[b] Number of moment conditions used in the empirical likelihood estimation procedure (see equation (17)).

[c] For each product, only estimates for $\tilde{q}_1, \ldots, \tilde{q}_{K-1}$ are reported; $\tilde{q}_K = 1 - \sum_{k=1}^{K-1} \tilde{q}_k$.

[d] Indifferent points Δ_k computed as $Ep_{(1:k)} - Ep_{(1:k+1)}$ (the expected price difference from having k versus $k+1$ price quotes), using the empirical price distribution. Including shipping and handling charges.

Estimation Results: Sequential Search Model

TABLE 3 Estimates of Sequential-Search Model

Product	δ_1	δ_2	Median[a] Search Cost	Selling Cost r	α[b]	$F_c^{-1}(1-\alpha;\boldsymbol{\theta})$	Log-L Value
Stokey-Lucas	.46 (.02)	1.55 (.03)	29.40 (1.45)	22.90 (1.31)	.58	19.19	31.13
Lazear	.40 (.01)	1.15 (.01)	16.37 (1.00)	11.31 (.79)	.69	4.56	34.35
Billingsley	.25 (.01)	2.01 (.04)	9.22 (.94)	65.37 (.83)	.51	8.43	23.73
Duffie	.21 (.02)	4.57 (.29)	10.57 (2.01)	28.24 (1.63)	.54	7.00	18.93

Note: Including shipping and handling charges. Standard errors in parentheses. δ_1 and δ_2 are parameters of the gamma distribution; see equation (13).

[a] As implied by estimates of the parameters of the gamma search-cost distribution.

[b] Proportion of consumers with reservation price equal to $\overline{p}$, implied by estimate of r (see equation (11)).

Hortacsu and Syverson (2004): Product Differentiation, Search Costs, and Competition

- One important limitation of the search models considered by Hong and Shum is that consumers sample stores **randomly**
- In particular sampling probabilities are independent of store characteristics and past purchase decisions
- The goal of Hortacsu and Syverson is to quantify the importance of search frictions, while accounting for **cost** and **quality** differences across products.
  - Differentiation and search cost affects both the distribution of prices, and the sampling (or matching) probability
  - More complicated identification problem...

Two Facts: Proliferation of Funds and Dispersion in Fees

TABLE I
PRICE DISPERSION WITHIN FUND SECTORS

Sector	N	Mean price	Coefficient of variation	75th to 25th Percentile ratio	90th to 10th Percentile ratio
Aggressive growth	1278	191.0	0.485	2.0	3.1
Balanced growth	472	164.2	0.439	2.2	3.7
High-quality bonds	862	118.1	0.566	2.5	4.9
High-yield bonds	337	167.3	0.387	2.1	3.2
Global bonds	358	182.3	0.402	2.0	3.5
Global equities	452	228.3	0.374	1.6	2.8
Growth and income	978	158.4	0.830	2.5	5.5
Ginnie Mae	182	144.0	0.460	2.4	4.0
Gov't securities	450	131.9	0.549	2.5	4.7
International equities	1267	225.5	0.432	1.9	3.2
Income	218	170.8	0.415	2.2	3.4
Long-term growth	1812	179.4	0.421	2.0	3.1
Tax-free money market	455	62.7	0.440	1.6	3.2
Gov't securities money market	437	59.5	0.611	1.8	4.8
High-quality muni bond	541	137.2	0.624	2.4	4.1
Single-state muni bond	1326	150.3	0.384	1.7	3.6
Taxable money market	541	79.2	0.726	2.0	7.1
High-yield money market	62	160.4	0.408	1.7	3.3
Precious metals	35	256.1	0.399	1.6	3.3
Sector funds	511	200.8	0.364	1.8	2.9
Total return	323	178.2	0.415	1.9	3.3
Utilities	94	182.8	0.359	1.7	3.2
Retail S&P 500 index funds	**82**	**97.1**	**0.677**	**3.1**	**8.2**

Model Overview

- Consumers have homogenous preferences over funds (vertical):

$$u_j = x_j\beta - p_j + \xi_j$$

- **Assumptions:**
  1. Matching probability for fund j: $\rho_j = \rho(Z_j)$
  2. Consumers know the distribution of utility: $H(u) = \frac{1}{n}\sum_j 1(u_j < u)$
  3. Search costs are linear and distributed according to a log-normal: $F_c(\cdot)$
  4. Each fund has a different marginal cost (common knowledge): mc_j
- **Notes:**
  - As in Varian, search frictions imply that **inferior** products have positive market shares
  - Price dispersion arises as a **pure strategy** Nash equilibrium because firms have different marginal cost.

Optimal Search Decision

- **Assumption:** Consumers search sequentially with replacement, and are allowed to "revisit" previously searched funds.
  - There are **always** n options to search at any point in time.
  - Ranking of options: $u_1 < u_2 < \cdots < u_n$.

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  - Ranking of options: $u_1 < u_2 < \cdots < u_n$.
- **Optimal search:** Continue searching if best alternative, u^*, is such that,

$$c_i \leq \int_{u^*}^{\bar{u}} (u - u^*) dH(u) = \text{Marginal benefit}$$

- Since the "search urn" is discrete, the optimal search problem yields n cutoffs.
- Consider a consumer with best alternative u_j who is indifferent between stopping and continuing:

$$c_j = \sum_{k=j}^{N} \rho_k (u_k - u_j)$$

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$$c_j = \sum_{k=j}^{N} \rho_k (u_k - u_j)$$

- In other words, c_j is the lowest possible search cost of any investor who purchases fund j in equilibrium.

Demand

- The model implies fixed cutoffs: High search cost consumers search more funds.
- However, since cutoffs differ across products, consumers will have different search **histories** depending of which fund they are matched with first.
- In particular, **higher quality** products (i.e. $u_k > u_{k-1}$) are associated with **lower** search cutoffs

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- We recursively construct market shares starting with the lowest quality product:
  - Only consumers with $c > c_1$ buy fund 1
  - This happens if those consumers are matched with the lowest-quality product first:

$$q_1 = \rho_1(1 - F_c(c_1))$$

Demand (continued)

- Similarly, the market share of product 2 includes consumers with $c > c_1$ who got "lucky" and sampled product 2 first (prob. ρ_2), and consumers with search cost $c_2 < c < c_1$ who sampled fund 2 first or second (prob. $\rho_2/1 - \rho_1$). This produces a demand curve:

$$q_2 = \rho_2(1 - G(c_1)) + \frac{\rho_2}{1 - \rho_1}[G_(c_1) - G(c_2)]$$

- Analogous linear equations can be constructed for the other products as function of $G(c_k)$.

Demand (continued)

- Similarly, the market share of product 2 includes consumers with $c > c_1$ who got "lucky" and sampled product 2 first (prob. ρ_2), and consumers with search cost $c_2 < c < c_1$ who sampled fund 2 first or second (prob. $\rho_2/1 - \rho_1$). This produces a demand curve:

$$q_2 = \rho_2(1 - G(c_1)) + \frac{\rho_2}{1 - \rho_1}[G_(c_1) - G(c_2)]$$

- Analogous linear equations can be constructed for the other products as function of $G(c_k)$.
- **Takeaway:** Given an assumption on the matching function $\rho(Z_j)$, the distribution of market shares allows us to recover the distribution of search-costs at n percentiles.

Price Competition

- **Bertrand-Nash:** Firms simultaneously choose prices to maximize profits,

$$\max_{p_j}(p_j - mc_j)q_j(p_j, p_{-j})$$

$$\text{FOC: } \frac{\partial q_j(p_j, p_{-j})}{\partial p_j}(p_j - mc_j) + q_j = 0$$

- **Note:** This FOC is very similar to the one used in pricing games with horizontal differential (e.g. Logit).
  - Market-power originates from information frictions and random-matching, instead of horizontal taste differences.

Mutual Fund Fees and Market Shares

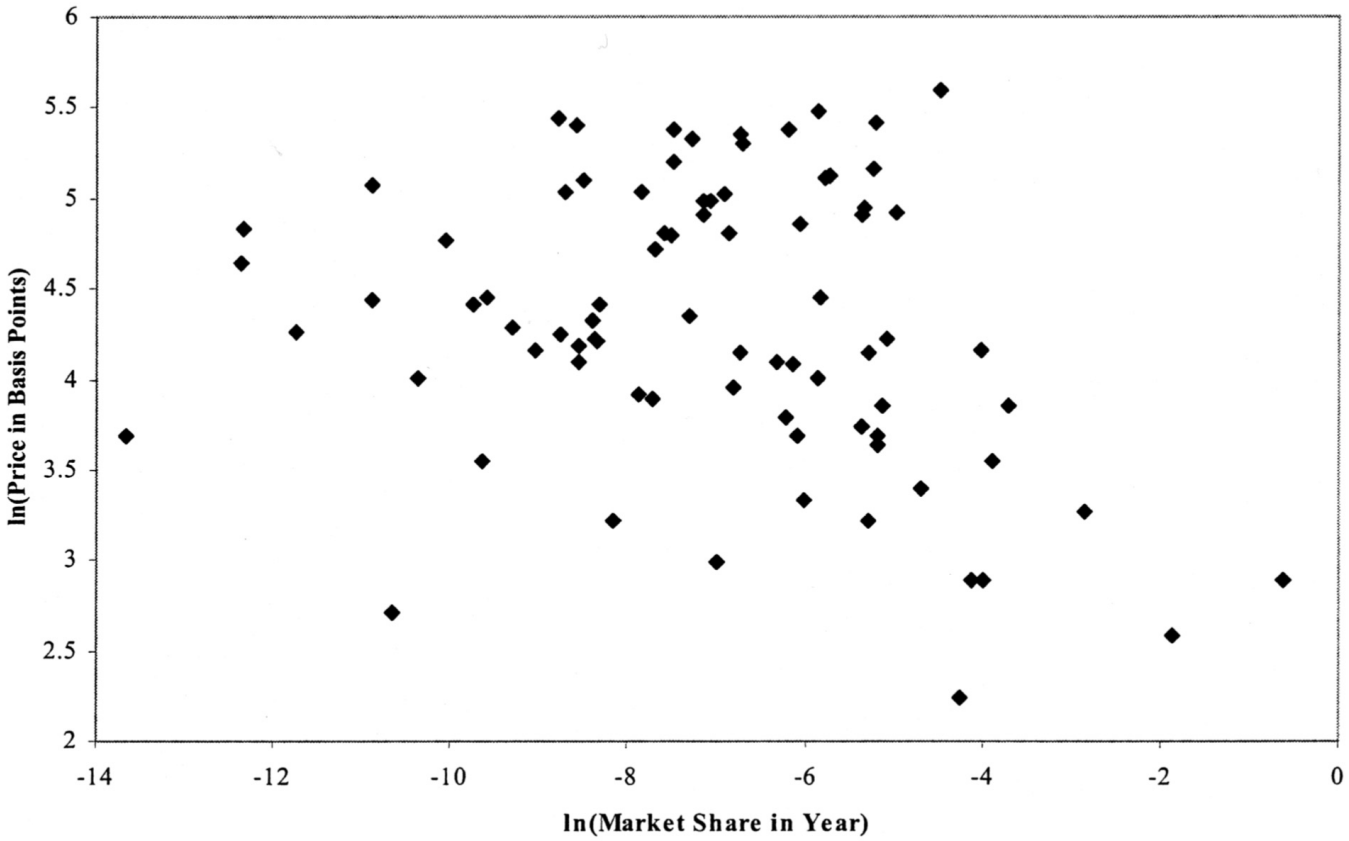


FIGURE III
Log Price Versus Log Market Share, 2000

- The fact the relationship between prices and market shares is non-monotonic rejects a simple model with homogenous funds (i.e. shares = search cost CDF)

TABLE III

SEARCH MODEL WITH UNEQUAL SAMPLING PROBABILITIES

Parameters	(A)	(B)	(C)	(D)	(E)
Log (mean search cost)	−6.17	−6.68	−6.58	−6.78	−6.33
	(0.06)	(0.06)	(0.13)	(0.26)	(0.11)
Variance of logged search costs	1.88	2.07	1.79	1.89	1.95
	(0.03)	(0.06)	(0.06)	(0.08)	(0.04)
Mean marginal cost, basis points	4	12	11	16	4
	(1)	(1)	(2)	(2)	(1)
α	2.62	2.43	2.58	2.44	3.06
	(0.04)	(0.03)	(0.03)	(0.09)	(0.04)
γ	—	—	—	—	0.11
					(0.03)
Time trend of mean search cost	−0.38	−0.50	−0.29	−0.51	−0.30
	(0.05)	(0.04)	(0.08)	(0.08)	(0.06)
Time trend of search cost variance	0.20	0.18	0.17	0.18	0.16
	(0.05)	(0.04)	(0.02)	(0.02)	(0.02)
R^2, prices	0.92	0.97	0.82	0.94	0.99
R^2, quantities	0.98	0.83	0.98	0.84	0.98
Median search cost (1996), b.p.	21	12	14	11	18
IQR of search cost range (1996), b.p.	5.9 to 75	3 to 50	4 to 46	3 to 41	4.7 to 67
Median search cost (2000), b.p.	5	2	4	1	5
IQR of search cost range (2000), b.p.	0.7 to 28	0.3 to 11	0.8 to 23	0.2 to 9	0.9 to 30

Result: Decline in Average Search Costs

Assumption: Uniform matching probability

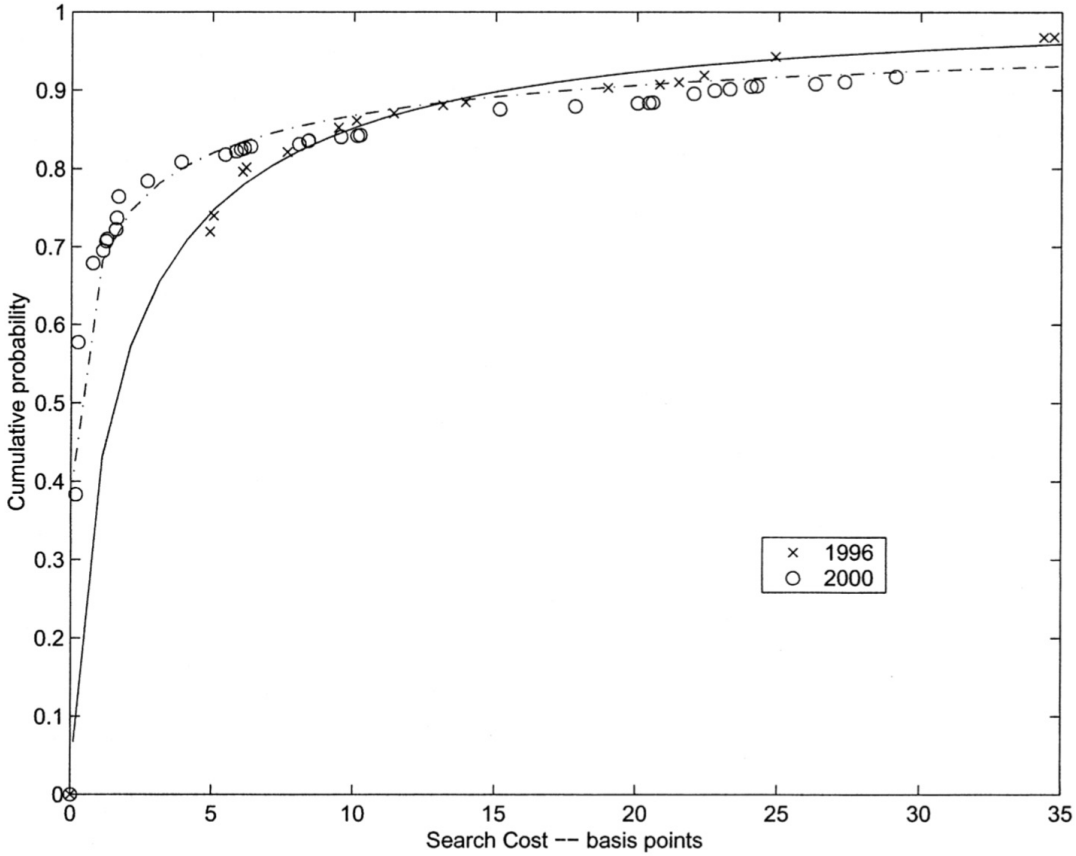


FIGURE IV

Estimated Search Cost Distributions, 1996 and 2000

The figure plots the c.d.f.s of the implied search cost distributions from the heterogeneous-funds model in subsection V.C. The plotted points are the nonparametrically identified cutoff search costs. The solid (dashed) line is a parametric lognormal distribution fit to the 1996 (2000) nonparametric cutoffs.

Gavazza (2016): Decentralized Asset Markets

- The previous models focussed on explaining the dispersion of prices
- Another important motivation to study search frictions is to measure the importance of *misallocation*
  - How long does it take for buyers to find sellers?
  - Are assets allocated efficiently based on valuations?
  - What is the role of intermediaries in alleviating trading/search frictions?

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- **Case study:** Business aircrafts market
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  - Intermediaries: 60% of transactions involve a dealer
  - Price dispersion: Large absolute deviation from Bluebook prices (17%)

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  - Intermediaries: 60% of transactions involve a dealer
  - Price dispersion: Large absolute deviation from Bluebook prices (17%)
- Modeling strategy:
  - *Matching friction:* Buyers are randomly matched with sellers ($\perp$ age)
  - *Bargaining:* Nash-Bargaining
  - *Intermediaries:* Dealers have more bargaining power, but have a higher matching probability

Preliminaries: Competitive Pricing with Search and Matching Frictions

- **Baseline model:** Mortensen and Wright (IER, 2002)
- **Model setup:**
  - Buyers: WTP $= x \sim F(x)$
  - Sellers: Cost $= y \sim G(y)$
  - Buyers/sellers leave the market after transaction
  - Inflow of new buyers and sellers: (b, s)
  - Stock of buyers and sellers (endogenous): (B, S)
  - Number of matches: λBS (i.e. $\lambda =$ poisson meeting probability)

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  - Stock of buyers and sellers (endogenous): (B, S)
  - Number of matches: λBS (i.e. $\lambda =$ poisson meeting probability)
- Steady-state competitive equilibrium (Walrasian):

$$b[1 - F(p^*)] = sG(p^*) = q^*$$

- In a **decentralized** market, prices in general will differ from p^* due to search frictions.

Pricing: Bargaining with Complete Information

- Consider a bilateral negotiation between (x, y), s.t. $x > y$
- If negotiation fails, the outside option of the buyer (seller) is the pesent value of searching for another seller (buyer): $V_b(x)$ and $V_s(y)$
- The continuation values determine the **surplus of the transaction**:
  - The buyer rejects offers if: $x - p < V_b(x)$ or $p > V_b(x) - x$
  - The seller rejects offers if: $p - y < V_s(y)$ or $p < V_s(y) + y$
  - Total surplus (gain from trade): $S(x, y) = x - y - V_b(x) - V_s(y)$

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  - Total surplus (gain from trade): $S(x, y) = x - y - V_b(x) - V_s(y)$
- **Nash-Bargaining:** Opportunity cost + Split of surplus

$$p(x, y) = y + V_s(y) + \beta S(x, y)$$

where β is the bargaining power of the seller.

Value Functions

- The value of becoming a **buyer** is given by:

$$\begin{aligned} rV_b(x) &= \lambda S \int \max\{x - p(x,y) - V_b(x), 0\} d\Gamma(y) - c_s \\ &= (1-\beta)\lambda S \int \max\{S(x,y), 0\} d\Gamma(y) - c_s \end{aligned}$$

- The value of becoming a **seller** is given by:

$$\begin{aligned} rV_s(y) &= \lambda B \int \max\{p(x,y) - y - V_s(y), 0\} d\Phi(x) - c_s \\ &= \beta\lambda B \int \max\{S(x,y), 0\} d\Phi(y) - c_s \end{aligned}$$

Entry and steady state

- Entry/exit decisions:
  - Buyer x enter if $rV_b(x) > 0$, or $x > r_b$
  - Seller y enter if $rV_s(y) > 0$, or $y < r_s$
- The reservation value are implicitly defined by two indifference conditions:

$$c_s = (1-\beta)\lambda S \int \max\{r_b - y - V_s(y), 0\} d\Gamma(y)$$

$$c_s = \beta\lambda B \int \max\{x - r_s - V_b(x), 0\} \Phi(x)$$

- The search equilibrium is defined by imposing additional steady-state conditions: The inflow and outflow of buyers/sellers are equalized (given thresholds (r_b, r_s)).

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- The search equilibrium is defined by imposing additional steady-state conditions: The inflow and outflow of buyers/sellers are equalized (given thresholds (r_b, r_s)).
- The efficiency of the equilibrium (relative to Walrasian) depends on:
  - Thickness of the market: B/S
  - Matching and search frictions: (λ, c_s)
  - Patience of buyers and sellers: r

Gavazza (2011): Empirical Evidence of Trading Frictions

- Before we extend the model to study the aircraft market, it is useful to look at the data see if trading frictions explain prices and trading patterns

Gavazza (2011): Empirical Evidence of Trading Frictions

Aircraft Price Dispersion

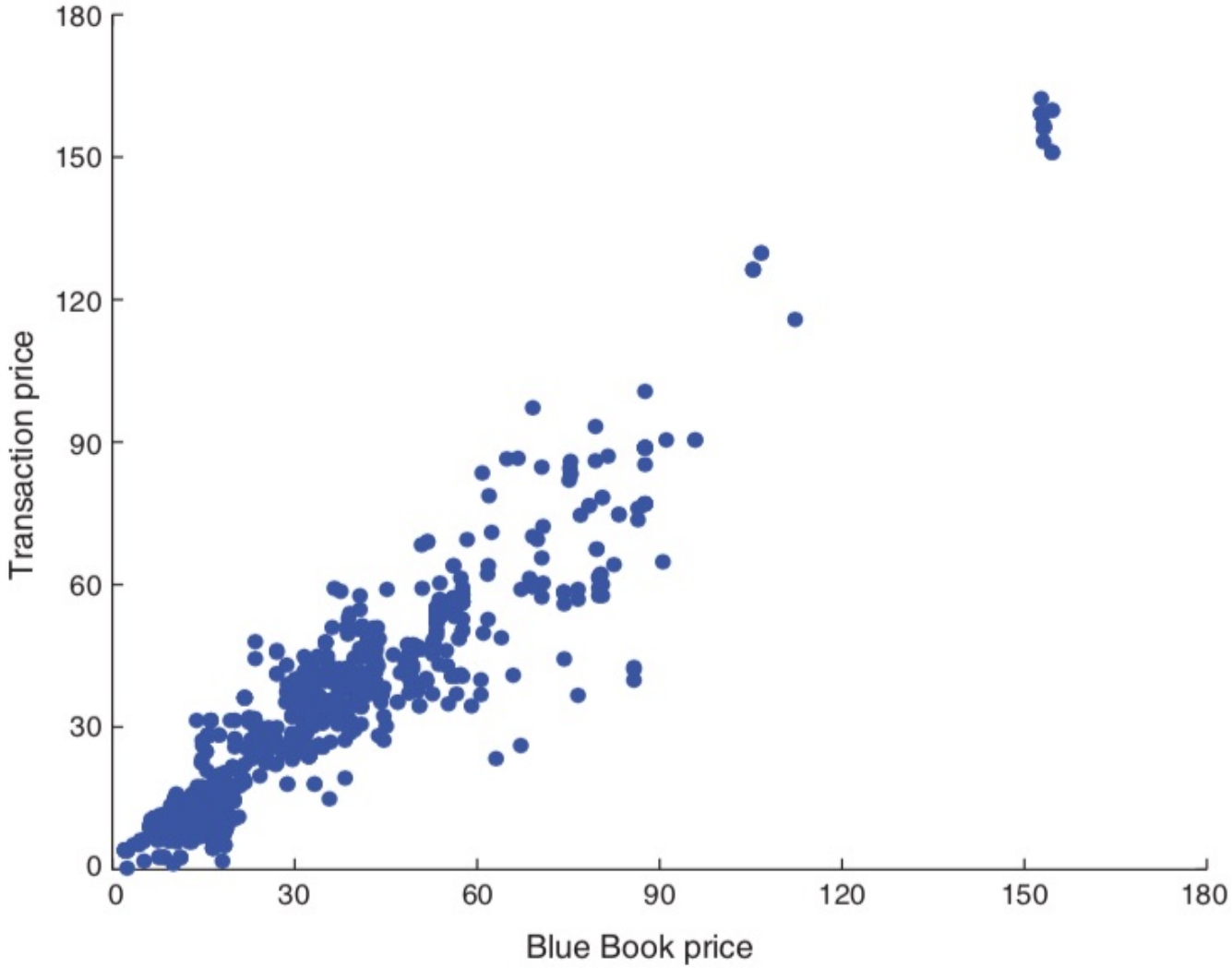


FIGURE 1. TRANSACTION PRICE VERSUS BLUE BOOK PRICE

Gavazza (2011): Empirical Evidence of Trading Frictions

Market Thickness and Trading Probability

TABLE 3—MARKET THICKNESS AND AIRCRAFT TURNOVER: HAZARD MODEL ESTIMATES OF TRADING

Hazard	(1)	(2)	(3)	(4)
Age at the beginning of spell	0.04451	0.04344	0.00027	−0.00015
	(0.00453)	(0.00448)	(0.00487)	(0.00479)
log (*Airtype*)	0.25517		0.19042	
	(0.09215)		(0.09835)	
log (*Optype*)		0.37810		0.33125
		(0.12095)		(0.13469)
log-Likelihood	−136,760.3	−136,704.9	−132,836.8	−132,793.8
Observations	241,622	241,622	241,622	241,622
Aircraft	17,398	17,398	17,398	17,398

Notes: This table presents maximum likelihood estimates of Cox proportional hazard models that investigate the effect of market thickness on the probability of trading aircraft. AGE AT THE BEGINNING OF THE SPELL is the age of the aircraft (in years) at the time of the acquisition. *Airtype* is the total stock of aircraft of a given aircraft type in a given year. *Optype* is the total number of carriers operating a given aircraft type in a given year. Standard errors clustered at the aircraft type–year level in parentheses. All equations allow the baseline hazard to vary by aircraft type. The equations estimated in all columns also contain year fixed effects (not reported). The equations estimated in columns 3 and 4 additionally contain carrier fixed effects (not reported).

Gavazza (2011): Empirical Evidence of Trading Frictions

Market Thickness and Capacity Utilization

TABLE 4—MARKET THICKNESS AND AIRCRAFT UTILIZATION (*Difference-GMM estimates*)

	Fly_{jkit} (1)	Fly_{jkit} (2)	$\log(fh_{jkit})$ (3)	$\log(fh_{jkit})$ (4)
Age	−0.01497	−0.01279	−0.03437	−0.03521
	(0.00232)	(0.00281)	(0.00383)	(0.00361)
log (*Airtype*)	0.09842		0.18996	
	(0.04861)		(0.08998)	
log (*Optype*)		0.08586		0.09105
		(0.02413)		(0.03845)
Observations	64,279	64,279	60,139	60,139
Aircraft	11,596	11,596	11,240	11,240

Notes: This table presents Arellano and Bond GMM estimates of the effect of market thickness on capacity utilization. The dependent variable in columns 1 and 2 is a binary indicator equal to one if the aircraft had positive flying hours in a given year, and zero otherwise. The dependent variable in columns 3 and 4 is the log of annual flying hours. The sample is restricted to aircraft with strictly positive flying hours in the specifications of columns 3 and 4. *Age* is the age of the aircraft. *Airtype* is the total stock of aircraft of a given aircraft type in a given year. *Optype* is the total number of carriers operating a given aircraft type in a given year. Standard errors clustered at the aircraft type–year level in parentheses. All equations also contain year fixed effects (not reported).

Gavazza (2011): Empirical Evidence of Trading Frictions

Market Thickness and Dispersion in Utilization

TABLE 5—MARKET THICKNESS AND DISPERSION OF CAPACITY UTILIZATION
(*Difference-GMM estimates*)

	$\left\lvert \frac{fh_{jkit} - \overline{fh}_{akit}}{\overline{fh}_{akit}} \right\rvert$	$\left\lvert \frac{fh_{jkit} - \overline{fh}_{akit}}{\overline{fh}_{akit}} \right\rvert$	$\left\lvert \frac{fh_{jkit} - \overline{fh}_{akit}}{\overline{fh}_{akit}} \right\rvert_{fh_{jkit}>0}$	$\left\lvert \frac{fh_{jkit} - \overline{fh}_{akit}}{\overline{fh}_{akit}} \right\rvert_{fh_{jkit}>0}$
	(1)	(2)	(3)	(4)
Age	0.03102	0.03717	0.01672	0.01687
	(0.00559)	(0.00521)	(0.00267)	(0.00271)
log (*Airtype*)	−0.22892		−0.17437	
	(0.09246)		(0.06521)	
log (*Optype*)		−0.10631		−0.05421
		(0.04163)		(0.02861)
Observations	63,886	63,886	60,139	60,139
Aircraft	11,582	11,582	11,240	11,240

Notes: This table presents Arellano and Bond's GMM estimates of the effect of market thickness on the dispersion of capacity utilization. The dependent variable is the absolute percentage deviation of flying hours of a given aircraft from the average flying hours of all aircraft of the same age and model in the same year. The equations estimated in specifications 3 and 4 use only observations with strictly positive flying hours. *Age* is the age of the aircraft. *Airtype* is the total stock of aircraft of a given aircraft type in a given year. *Optype* is the total number of carriers operating a given aircraft type in a given year. Standard errors clustered at the aircraft type–year level in parentheses. All equations also contain year fixed effects (not reported).

Gavazza (2011): Empirical Evidence of Trading Frictions

Market Thickness and Price Dispersion

TABLE 7—MARKET THICKNESS AND AIRCRAFT PRICE DISPERSION: IV ESTIMATES

$\left\lvert \frac{p_{jkit} - \overline{p}_{akit}}{\overline{p}_{akit}} \right\rvert$	(1)	(2)	(3)	(4)
Age	0.00179	0.00265	0.00285	0.00393
	(0.00365)	(0.00377)	(0.00595)	(0.00618)
log (*Airtype*)	−0.13635		−0.15692	
	(0.04372)		(0.04116)	
log (*Optype*)		−0.10401		−0.11208
		(0.04365)		(0.04204)
R^2	0.419	0.413	0.494	0.486
Observations	1,570	1,570	1,570	1,570

Notes: This table presents IV estimates of the effect of market thickness on the dispersion of transaction prices. The dependent variable is the absolute percentage deviation of the transaction price of a given aircraft from the Blue Book price of an aircraft of the same age and model in the same year. *Age* is the age of the aircraft. *Airtype* is the total stock of aircraft of a given aircraft type in a given year. *Optype* is the total number of carriers operating a given aircraft type in a given year. Standard errors clustered at the aircraft type–year level in parentheses. All equations also contain year fixed effects (not reported). The equation estimated in columns 1 and 2 also contains aircraft-model, year, and noise-stage fixed effects (not reported). The equation estimated in columns 3 and 4 also contains aircraft-model, year, noise-stage, and engine-type fixed effects (not reported).

Gavazza (2016): An empirical equilibrium model of a decentralized asset market

- **Model setup:** (abstracting away from dealers)
  - Two productivity types: $z_h > z_l$
  - μ new agents enter with type z_h
  - Transition probability: $\Pr(z_{t+1} = z_l | z_h) = \lambda$ (absorbing)
  - Aircraft are differentiated based on age:
    - ★ Depreciation: $\partial\pi(z, a)/\partial a < 0$
    - ★ Complementarity: $\partial\pi(z_h, a)/\partial a < \partial\pi(z_l, a)/\partial a$

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    - ★ Complementarity: $\partial\pi(z_h, a)/\partial a < \partial\pi(z_l, a)/\partial a$
- **Matching:**
  - Poisson probability of a match: γ
  - Buyers and sellers are randomly matched:
    - ★ Arrival rate of a seller of type a for buyers: $\gamma_b(a) = \gamma\mu_s(a)$
    - ★ Arrival rate of a buyer for a seller is: $\gamma_s = \gamma\mu_b$
  - Number of matches of type a (increasing return): $\gamma\mu_b\mu_s(a)$

Value Functions

- The value of owning/using an aircraft of age a is given by:

$$rU_o(a, z_h) = \pi(z_h, a) + \lambda(V_o(a, z_l) - U_o(a, z_h)) + U'_o(a, z_h)$$

$$rU_o(a, z_l) = \pi(z_l, a) + U'_o(a, z_h)$$

where $V_o(a, z_l) = \max\{U_o(a, z_l), S_o(a, z_l)\}$.

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$$rU_o(a, z_h) = \pi(z_h, a) + \lambda(V_o(a, z_l) - U_o(a, z_h)) + U_o'(a, z_h)$$

$$rU_o(a, z_l) = \pi(z_l, a) + U_o'(a, z_h)$$

  where $V_o(a, z_l) = \max\{U_o(a, z_l), S_o(a, z_l)\}$.

- The value of agent of type z_h trying to sell an asset a is:

$$\begin{aligned} rS_o(a, z_h) &= \pi(z_h, a) - c_s + \lambda(V_o(a, z_l) - S_o(a, z_h)) \\ &\quad + \gamma_s \max\{p(a) + V_n(z_h) - S_o(a), 0\} + S_o'(a) \end{aligned}$$

  where $p(a)$ is the NB price, and $V_n(z) = \max\{U_n(z), S_n(z)\}$.

Value Functions (continued)

- The value of agents with no aircraft who are actively trying to buy is:

$$\begin{aligned} rS_n(z_h) &= -c_s + \lambda(V_n(z_l) - S_n(z_h)) \\ &\quad + \int \gamma_b(a)\{V_o(a, z_h) - p(a) - S_n(z_h), 0\}da \\ rS_n(z_l) &= -c_s + \int \gamma_b(a)\{V_o(a, z_l) - p(a) - S_n(z_l), 0\}da \end{aligned}$$

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- Transaction surplus and price between seller (z_l, a) and buyer z_h:

$$\text{Surplus}(a) = \underbrace{(U_o(a, z_h) - S_n(z_h))}_{\text{WTP}} - \underbrace{(S_o(a, z_l) - V_n(z_l))}_{\text{Opportunity cost}}$$

$$\Rightarrow p(a) = S_o(a, z_l) - V_n(z_l) + \theta\text{Surplus}(a)$$

Value Functions (continued)

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$$\Rightarrow p(a) = S_o(a, z_l) - V_n(z_l) + \theta\text{Surplus}(a)$$

- Notes:
  - Since firms are symmetric, trade will take place only between z_l (sellers) and z_h (buyers)
  - Because of depreciation, the gains from trade are larger for younger assets: older aircrafts will be scrapped or sold

Policy functions

- **High-types:**
  - Non-owners are always on the market looking to buy:

$$\text{Surplus}(a) \geq 0 \text{ if } a \leq a^*_{hn}$$

$$V_n(z_h) = S_n(z_h)$$

  - Owners either keep their asset, or scrap it when it is too old:

$$V_o(a, z_h) = \begin{cases} U_o(a, z_h) & \text{If } a < a^*_{ho} \\ S_n(z_h) & \text{If } a \geq a^*_{ho} \end{cases}$$

  - Trade frictions imply that agents keep their asset too long: $a^*_{ho} > a^*_{hn}$.

Policy functions

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  - Trade frictions imply that agents keep their asset too long: $a^*_{ho} > a^*_{hn}$.
- **Low-types:** Sell younger assets to realize gains from trade and scrap older assets (exit)

$$V_o(a, z_l) = \begin{cases} S_o(a, z_l) & \text{If } a < a^*_l \\ U_o(a, z_l) & \text{If } a \leq a^*_l < T \\ V_n(z_l) = 0 & \text{If } a = T \end{cases}$$

where $a^*_l \leq a^*_{hn}$ (i.e. sellers sell assets that buyers are willing to buy).

Distribution of Agents

- Distribution of high-types owners:
  - Initial condition: $\mu_{ho}(0) = x$
  - Transition:
$$\dot{\mu}_{ho}(a) = \gamma_b(a)\mu_{hn} - \lambda\mu_{ho}(a) \text{ if } a < a^*_{ho}$$
  - Terminal condition: $\mu_{ho}(a) = 0$ if $a^*_{ho} \leq a \leq T$.

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  - Terminal condition: $\mu_{ho}(a) = 0$ if $a^*_{ho} \leq a \leq T$.
- Distribution of low-types owners:
  - Initial condition: $\mu_{lo}(0) = 0$
  - Transition:
$$\dot{\mu}_{ho}(a) = \lambda\mu_{ho}(a)1(a < a^*_{h0}) - \gamma_s\mu_{lo}(a)1(a < a^*_l)$$

Distribution of Agents

- Distribution of high-types owners:
  - Initial condition: $\mu_{ho}(0) = x$
  - Transition:
$$\dot{\mu}_{ho}(a) = \gamma_b(a)\mu_{hn} - \lambda\mu_{ho}(a) \text{ if } a < a^*_{ho}$$
  - Terminal condition: $\mu_{ho}(a) = 0$ if $a^*_{ho} \leq a \leq T$.
- Distribution of low-types owners:
  - Initial condition: $\mu_{lo}(0) = 0$
  - Transition:
$$\dot{\mu}_{ho}(a) = \lambda\mu_{ho}(a)1(a < a^*_{h0}) - \gamma_s\mu_{lo}(a)1(a < a^*_l)$$
- Distribution of non-owners:
  - Transition of high-types:
$$\dot{\mu}_{hn} = (\mu - x) + \mu_{ho}(a^*_{ho}) - \lambda\mu_{hn} - \mu_{hn}\int_0^{a^*_l} \gamma_b(a)da$$
  - Stock of low-types:
$$\mu_{ln} = \lambda\mu_{hn} + \gamma_s\int_0^{a^*_l} \mu_{lo}(a)da + \mu_{lo}(T)$$

Steady-state equilibrium conditions

1. Constant mass of owners and non-owners:

$$\mu_{hn} \text{ and } \int_0^T \mu_i(a)da, \quad \forall i = ho, lo$$

2. Mass of exiters equals the mass of new entrants: $\mu_{ln} = \mu$
3. Number of high-value agents: $\mu_{hn} + \int_0^T \mu_{ho}(a)da = \frac{\mu}{\lambda}$.
4. Number of active asset: $x = \mu_{ho}(a) + \mu_{lo}(a)$ if $a \leq a^*_{ho}$.

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- These conditions are used to solve for the endogenous mass of agents in each state as a function of the parameters:

$$\mu, x, T, \lambda, \gamma, \pi(\cdot)$$

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- **Policy function:**
  - **Sellers:** Dealer-Owners sell assets younger than a^*_{do} (scrap older)
  - **Buyers:** Non-owners try to buy assets younger than a^*_{dn}
  - Due to frictions: $a^*_{dn} < a^*_{do}$

Estimation: GMM

- **Calibrated parameters:** ρ (discount) and $T = 160$ quarter
- **Parameters to estimate:** $\psi = \{\lambda, \gamma_s, \gamma_{sd}, \alpha_{ds}, z_h, z_l, \delta, c_s, \theta_s, \theta_d\}$
  - **Note:** γ_s and γ_{sd} are endogenous parameters that are recovered from data on the fraction of retail-to-retail transactions, and detail-to-retail transaction.
- **Moments:**
  - Fraction of aircraft for sale
  - Fraction of aircraft for sale by dealers
  - Fraction of retail-to-retail transactions
  - Fraction of dealer-to-retail transactions
  - Average age of aircraft for sale
  - Hedonic price coefficients (indirect-inference moments):

$$p(a) = \beta_0 + \beta_1 \exp(-\beta_2 a)$$

$$p_B(a) = \beta_3 + \beta_4 \exp(-\beta_5 a)$$

Estimation Results

TABLE II

ESTIMATES[a]

λ	0.0457	z_h	695,610
	(0.0035)		(41,258)
γ_s	0.1825	z_l	114,060
	(0.0015)		(16,250)
γ_{sd}	0.1737	c_s	1,622.7
	(0.0447)		(486.23)
α_{ds}	0.7021	θ_s	0.3925
	(0.0054)		(0.0923)
δ_2	0.0167	θ_d	0.9732
	(0.1346×10^{-3})		(0.0054)
γ	5.9717×10^{-5}	μ	482.3
	$[3.823; 6.225] \times 10^{-5}$		[475.7; 733.0]
γ'	2.2973×10^{-4}	k	244,767
	$[1.444; 2.333] \times 10^{-4}$		[232,810; 303,120]

[a]This table reports the estimates of the parameters. Asymptotic standard errors, in parentheses. 95-percent confidence intervals in brackets are obtained by bootstrapping the data using 100 replications.

Counterfactual Analysis: Zero search cost

TABLE IV

COMPARISON WITH WALRASIAN MARKET[a]

	Estimated Model (1)	Walrasian Market (2)	$c_S = 0$ (3)
$\int_0^T (\mu_{lo}(a) + \mu_{do}(a))\,da$	1,681 [1,646; 2,399]	0 [0; 0]	1,698 [1,683; 2,480]
$\frac{\int_0^T (\mu_{lo}(a)+\mu_{do}(a))\,da}{A}$	0.183 [0.179; 0.256]	0 [0; 0]	0.183 [0.182; 0.264]
$\int_0^{a_l^*} \mu_{lo}(a)\,da + \int_0^{a_{do}^*} \mu_{do}(a)\,da$	1,215 [1,202; 1,755]	0 [0; 0]	1,218 [1,152; 1,742]
$\frac{\int_0^{a_l^*} \mu_{lo}(a)\,da+\int_0^{a_{do}^*} \mu_{do}(a)\,da}{A}$	0.132 [0.130; 0.187]	0 [0; 0]	0.132 [0.125; 0.184]
$p(0)$	17,611,151 $[17{,}253; 17{,}969] \times 10^3$	21,812,709 $[21{,}532; 22{,}145] \times 10^3$	17,648,470 $[17{,}249; 18{,}097] \times 10^3$
$p(10)$	8,437,910 $[8{,}366; 8{,}531] \times 10^3$	11,008,194 $[10{,}633; 11{,}221] \times 10^3$	8,475,461 $[8{,}148; 8{,}672] \times 10^3$
Allocations	2.054×10^9 $[1.942; 2.060] \times 10^9$	2.371×10^9 $[2.324; 2.397] \times 10^3$	2.057×10^9 $[1.941; 2.062] \times 10^3$
Search Costs	6.534×10^6 $[4.817; 14.182] \times 10^6$	0 [0; 0]	0 [0; 0]
Dealers Costs	244×10^6 $[232.8; 303.1] \times 10^6$	0 [0; 0]	255×10^6 $[206.5; 311.6] \times 10^6$
Welfare	1.802×10^9 $[1.614; 1.803] \times 10^9$	2.371×10^9 $[2.324; 2.397] \times 10^3$	1.801×10^9 $[1.632; 1.828] \times 10^3$

[a]This table reports counterfactual allocations and prices. 95-percent confidence intervals in brackets are obtained by bootstrapping the data using 100 replications.

Counterfactual Analysis: The role of dealers

TABLE V

COMPARISON WITH A MARKET WITH NO DEALERS[a]

	Estimated Model	No Dealer Market
$\int_0^T \mu_{lo}(a)\,da + \int_0^{a^*_{do}} \mu_{do}(a)\,da$	1,681 [1,646; 2,399]	2,013 [1,979; 2,634]
$\frac{\int_0^T \mu_{lo}(a)\,da + \int_0^{a^*_{do}} \mu_{do}(a)\,da}{A}$	0.183 [0.179; 0.256]	0.206 [0.203; 0.270]
$\int_0^{a^*_l} \mu_{lo}(a)\,da + \int_0^{a^*_{do}} \mu_{do}(a)\,da$	1,215 [1,202; 1,755]	2,000 [1,979; 2,616]
$\frac{\int_0^{a^*_l} \mu_{lo}(a)\,da + \int_0^{a^*_{do}} \mu_{do}(a)\,da}{A}$	0.132 [0.130; 0.187]	0.205 [0.203; 0.268]
$p(0)$	17,611,151 $[17{,}253; 17{,}969] \times 10^3$	18,118,803 $[17{,}791; 18{,}738] \times 10^3$
$p(10)$	8,437,910 $[8{,}366; 8{,}531] \times 10^3$	9,086,250 $[8{,}836; 9{,}295] \times 10^3$
Allocations	2.054×10^9 $[1.942; 2.060] \times 10^9$	1.984×10^9 $[1.859; 1.990] \times 10^9$
Search Costs	6.534×10^6 $[4.817; 14.182] \times 10^6$	7.817×10^6 $[5.631; 16.368] \times 10^6$
Dealers Costs	244×10^6 $[232.8; 303.1] \times 10^6$	0 [0; 0]
Welfare	1.802×10^9 $[1.614; 1.803] \times 10^9$	1.977×10^9 $[1.844; 1.983] \times 10^9$

[a]This table reports counterfactual allocations and prices in a market without dealers. 95-percent confidence intervals in brackets are obtained by bootstrapping the data using 100 replications.

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